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XIX. *On a Modified Water-dropping Influence-machine.* *By* Professor SILVANUS P. THOMPSON, *D.Sc.**

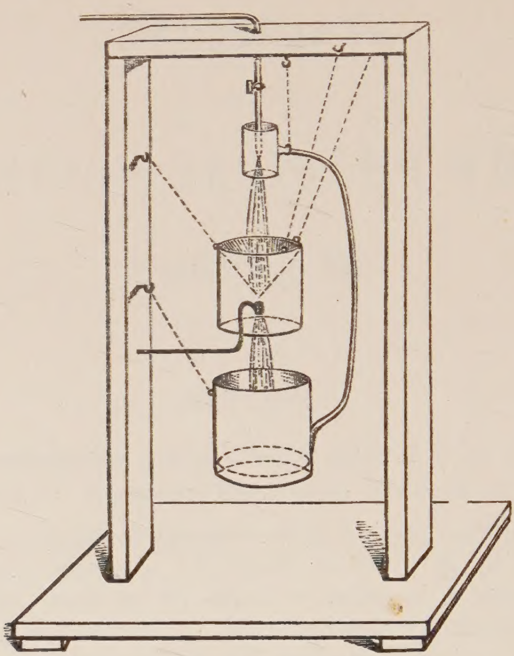
THE ordinary form of water-dropping influence-machine, as devised by Sir William Thomson †, possesses some inconveniences: it requires a double jet of water and special arrangements for high insulation. A simpler form, requiring but one water-jet and mere silk strings (well paraffined) as insulators, has been found by the author to give far less trouble, and to work well for lecture demonstrations.

From a wooden frame are hung by silk strings three simple metal vessels, the highest and lowest being rigidly connected together with a stiff metal wire. The highest is a small cylinder open at both ends; the lowest is an open pot which receives the water. The intermediate vessel is open at the bottom; and is provided at the top with a funnel, the upper rim of which is soldered inside the lip of the cylinder, and its depth such that its central aperture is about at the middle of the cylinder. An insulated wire, recurved as shown, is carried up clear under the funnel in the middle vessel, and should touch the drops as they fall below the aperture. The

* Read January 28, 1888.

† Proc. Roy. Soc. June 20, 1867; and Reprint of Papers on Electrostatics, p. 321.

water-jet, which must have a fine orifice, is inserted about half-way into the uppermost vessel. A single point of water will



suffice to gather a plentiful charge. To watch the process of charging, two gold-leaf electroscopes may be connected respectively to the middle and to the lowest vessels.

The same arrangement will answer for sand-dropping if a second, uninsulated funnel to contain the sand be provided above the topmost cylinder, and arranged with its lower end entering into the cylinder, so that the jet of sand breaks away from the orifice at the proper height. As dry sand is a very bad conductor, the apparatus is found to work with greater certainty if the sand is previously agitated with finely powdered plumbago, or with some sufficiently adherent metallic powder, such as the finer qualities of Bessemer bronze.

XX. *The Effect of Magnetization on the Thermoelectrical and other Physical Properties of Bismuth.* By HERBERT TOMLINSON, *B.A.**

IN a paper read before the Royal Society on January 26, 1882†, the author has given an account of an experiment relating to the effect of longitudinal magnetization on the electrical resistance of bismuth. The subject has since been taken up much more fully by Righi ‡, Leduc §, Hurion ‖, and Albert v. Ettingshausen and Walther Nernst ¶. In the author's paper quoted above are also described experiments on the effects of magnetization on the electrical resistance of iron, steel, nickel, and cobalt, the results of which are summarized in the following table:—

TABLE I.

Metal.	Condition.	Increase of resistance per unit produced by a C.G.S. unit of magnetizing force, $\frac{\Delta r}{rM_f}$.	Magnetic susceptibility. κ**.	$\frac{\Delta r}{rM_f}\Big/\kappa$**.
Iron	Annealed.	2335×10^{-8}.	30	$0{\cdot}8\times10^{-6}$
Steel	Annealed.	1500 „		
Steel	Unannealed.	1137 „		
Steel	Very hard.	70 „		
Nickel......	Annealed.	8070 „	8·8	9·2 „
Nickel......	Unannealed.	4343 „		
Cobalt. ...	Unannealed.	638 „	4·4	1·4 „
Bismuth ...	Unannealed.	21 „	−·000014††	1500 „
Copper ...	Annealed.	0‡‡ „		

* Read January 28, 1888.

† "The Influence of Stress and Strain on the Physical Properties of Matter," Phil. Trans. vol. clxxiv. (1883, part 1). Abstract, Proc. Roy. Soc. No. 218 (1882).

‡ *Acc. R. dei Lincei,* 1883, 1884.

§ *Bull. de la Soc. française de Phys.* 1884.

‖ *Comptes Rendus,* 1884, 1885.

¶ *Sitzb. der kais. Akad. der Wissensch.* 1887.

** The values of κ were determined for the same magnetizing forces as those used for producing alteration of resistance.

†† Taken from Von Ettingshausen's determinations (*Wien. Ber.* 1882).

‡‡ No change amounting to $\frac{1}{4000000}$ could be detected with a magnetizing force of 90 C.G.S. units.

The data in this table refer to the effects of *temporary* magnetization; and in the case of iron, steel, and nickel, represent only verifications and extensions of the labours of previous observers. Abraham, Edlund, Mousson, and Wartmann all made search for magnetic alteration of the resistance of iron. W. Thomson * seems, however, to have been the first to arrive at any definite result. He found the resistance to be increased along the lines of magnetization and decreased across them. W. Thomson has been followed by Beetz †, Tomlinson ‡, Chwolson §, Auerbach ‖, and De Lucchi ¶. These have all confirmed the results of Thomson so far as longitudinal magnetization is concerned; but Beetz failed to obtain anything but negative results with transverse magnetization, and attributed the decrease of resistance observed by Thomson to mere mechanical pull. The author has, however, pointed out ** the improbability of this last supposition. W. Thomson had also previously proved †† that the electrical resistance of nickel is increased to a greater extent than that of iron by longidinal magnetization; whilst Faé ‡‡ has recently verified the author's result concerning cobalt. Lastly, Goldhammer §§ has published a comparative study of the three paramagnetic metals—iron, cobalt, and nickel, and of the three diamagnetic metals—bismuth, antimony, and tellurium ‖‖.

There are several points in the table given above to which it is desirable to direct attention. In the first place the resistance of all the metals is *increased* by longitudinal magnetization. In the second it by no means always follows that the metals which possess the greatest magnetic susceptibility are those which are most affected in their conductivity by a given amount of magnetizing-stress. We see, for instance, from the third and fourth columns, that whilst the value of $\frac{\Delta r}{r M_f}$ for

* "Electrodynamic Qualities of Metals," Phil. Trans. 1856, Part iv.

† Pogg. *Ann.* vol. cxxviii. (1886).

‡ Proc. Roy. Soc. vol. xxiii. (1875). Also *loc. cit.*

§ Carl's *Rep.* vol. xiii. (1877).

‖ Phil. Mag. vol. viii. (1879).

¶ *Atti del R. Ist. Veneto*, viii. (1882).

** *Loc. cit.* pp. 165, 166.

†† Proc. Roy. Soc. vol. viii. (1857).

‡‡ *Atti del R. Ist. Veneto*, 1887.

§§ Wied. *Ann.* xxxi. (1887).

‖‖ See also a memoir, entitled "An Experimental Study of the Influence of Magnetism and Temperature on the Electrical Resistance of Bismuth and its Alloys with Lead and Tin," by Edmond van Aubel. *Ante*, p. 124.

nickel is twice that for iron, the magnetic susceptibility of iron is between three and four times as great as that of nickel. It is, however, when we come to consider the fifth column that we meet with the most remarkable differences in the effects of magnetization. The numbers in this column represent the increase of resistance per unit which would be produced in each of the metals by magnetizing them to such an extent that each cubic centimetre would possess unit magnetic moment. The effect of the magnetization in this case would be nearly twice as great for cobalt as for iron, twelve times as great for nickel as for iron, and, speaking very roughly, *two thousand* times as great for bismuth as for iron. Startling as this last result is, it sinks into insignificance when contrasted with the results obtained by other observers. The effect of magnetization on the electrical resistance of bismuth is largely influenced, amongst other things, by the amount of impurity in the metal and the magnitude of the magnetizing force. Thus, from Ettingshausen's researches, from which Table II. has been compiled, we learn that the value of $\frac{\Delta r}{r\mathrm{M}_f}$ for pure bismuth and for very large magnetizing forces* may become nearly 200 times as great as

TABLE II.

Transverse Magnetization.

Magnetizing force, in C.G.S. units.	Increase of resistance per unit produced in pure bismuth by a C.G.S. unit of magnetizing force, $\frac{\Delta r}{r\mathrm{M}_f}$.	Ditto in bismuth alloyed with one per cent. of tin.
1600	1610×10^{-8}	
3160	2490 „	
5880	3350 „	
8410	3660 „	660×10^{-8}
10470	3840 „	
11200	3890 „	

* The bismuth was acted upon by a transverse magnetizing force, which, however, has been proved to *increase* the resistance, though to a greater extent than a longitudinally magnetizing force.

that observed by the author*, with the comparatively low magnetizing force of 130 C.G.S. units; whilst the introduction of one per cent. of tin diminishes the value to *one sixth.* Provided that the susceptibility of bismuth be the same for very high magnetizing forces as for low ones, it would follow that if a bar of pure bismuth could be magnetized to unit intensity† its resistance would be nearly trebled, whilst the increase of resistance per unit would be at least 300,000 times as great as the corresponding change of resistance in iron. Considerations such as the above render it difficult to believe that what is observed in bismuth in such experiments as these is a real change in the specific resistance of the metal; and even with iron, nickel, and cobalt there seems to be evidence that the *whole* of the observed change produced by magnetization is not produced by mere rotation of the molecules. In the author's experiments on iron and nickel he found that the increase of resistance could be very closely represented by the formula

$$\frac{\Delta r}{r} = a\mathrm{M}_f + b\mathrm{M}\ ;$$

where $\frac{\Delta r}{r}$ denotes the increase of resistance per unit, and M_f and M_i are the magnetizing force and the magnetic intensity respectively. From this formula it follows that, even if the magnetizing force were so high that the ratio of increase of intensity to increase of force was extremely small, the resistance would nevertheless go on increasing very perceptibly indeed with the force. The values of the coefficient b were not very different in the two metals nickel and iron; but the coefficient a in nickel was about five times as great as in iron, and was nearly double the coefficient b. It would be a matter of considerable interest to ascertain whether it would really happen that, when very great magnetizing forces were employed, the resistance of nickel would go on increasing very perceptibly with the force‡.

* The specimen of bismuth used by the author has been, through the kindness of Professor J. M. Thomson, analyzed at the chemical laboratory at King's College, London; it contains 98·48 per cent. of bismuth.

† This would require a magnetizing force of 71,000 C.G.S. units.

‡ The magnitudes of the forces used by the author never exceeded 200 C.G.S. units.

If, however, on the one hand there is a difficulty in conceiving how magnetization can so largely affect the true specific resistance of bismuth, there is also an equal difficulty in accounting otherwise for what is observed. Hall's phenomenon cannot certainly be credited in any of the experiments which have yet been made with anything but a small fraction of the whole observed effect. It is true that bismuth has been found by Righi and others to have a very large rotational coefficient as compared with iron, nickel, or cobalt; but Ettingshausen and Nernst have shown* that whilst, with bismuth, antimony, and tellurium, the increase per unit of resistance produced by a magnetizing force of 7660 C.G.S. units is 0·20, 0·006, and 0·0014 respectively, the corresponding values of the rotatory power are −4·7, +0·18, and +790.

Neither, again, can the change of dimensions produced by magnetization in any of the metals be accountable for the increase of resistance. For though, curiously enough, loading an iron wire increases the resistance, and magnetizing it longitudinally increases the length, whilst loading a nickel wire† *decreases* the resistance and magnetization *decreases* the length, yet, according to Joule and others, when an iron wire is loaded to a certain extent longitudinal magnetization begins to decrease the length; whereas the author has shown that, even when iron is loaded nearly to breaking, longitudinal magnetization always produces increase of resistance. Besides, the changes of dimensions in nickel, iron, and bismuth produced by magnetization are far too small‡. Here again, however, it should be noticed that both the decrease of length produced by magnetization and the decrease of resistance produced by loading a nickel wire are considerably greater than the corresponding increase in the case of iron.

Whatever it is that causes magnetization to produce so large an effect on the electrical conductivity of bismuth, causes it to produce also a large effect on some of the other physical properties. The thermal conductivity of bismuth is, according to Leduc§ and Righi ‖, decreased both by longitudinal and

* *Loc. cit.* † See Phil. Trans. 1883.

‡ Prof. Barrett failed to detect any change produced in the dimensions of bismuth by magnetization.

§ *Comptes Rendus*, 1887. ‖ *Ibid.*

transverse magnetization by an amount which is about equal to the amount of decrease produced in the electrical conductivity; and though it would appear, from Ettingshausen's observations*, that the decrease of thermal conductivity is decidedly less than the decrease of electrical conductivity, yet even this observer makes the former comparable with the latter; and we shall now see that the thermoelectrical properties of bismuth are quite as largely affected by magnetization as either the thermal or the electrical conductivity.

The Thermoelectrical Properties of Bismuth.

Sir W. Thomson has shown † that iron longitudinally magnetized is negative, and transversely magnetized positive, to iron unmagnetized. Barus and Strouhal ‡ have also investigated with great completeness the influence of magnetization on the thermoelectrical properties of steel of different tempers. Finally, Ewing has entered very fully § into the changes effected by longitudinal magnetization in iron when under different amounts of longitudinal stress. Thomson has also shown ‖ that nickel is rendered by longitudinal magnetization thermoelectrically positive to unmagnetized nickel; whilst the author has found ¶ cobalt when under longitudinal magnetization to be negative to the unmagnetized metal.

The experiment now about to be described was made nearly at the same period as the experiment on the effect of magnetization on the electrical resistance of bismuth and with the same bar. This bar was 25 centim. long and 0·33 centim. in diameter; it was placed in the axis of a magnetizing solenoid, S, specially constructed to avoid imparting heat to the magnetized core**; a preliminary examination proved that there was certainly no error arising from this cause. The arrangements are sufficiently shown in fig. 1, where S is the solenoid and A B the bar. The bar was encircled by two little air-chambers, C and D, through one of which steam was

* *Annalen der Physik und Chemie*, Band xxxiii. (1888).

† "Electrodynamic Qualities of Metals," Phil. Trans. Part iv. 1856.

‡ Bulletin of the U. S. Geological Survey, No. 14 (1885).

§ Phil. Trans. Part ii. 1886.

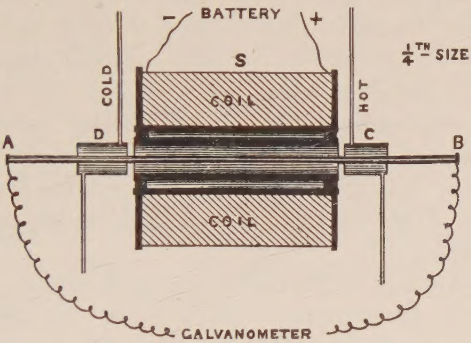
‖ *Loc. cit.*

¶ Proc. Roy. Soc. No. 241 (1885).

** For a description of this solenoid see Phil. Trans. 1883, *loc. cit.*

passed, and through the other water at a temperature of about 16° C. The ends A and B were connected by copper wires

Fig. 1.



with a very sensitive Thomson's reflecting-galvanometer, and were well buried in sawdust.

Since a bar of bismuth can never be obtained in a perfectly homogeneous condition throughout its whole length, there was a considerable thermoelectrical current already in existence before magnetization and the spot of light was driven off the scale. The light could be again brought on the scale by putting the adjusting-magnet low down; but this of course materially diminished the sensibility of the galvanometer, and as the effect to be looked for was likely to be very small this was not desirable. Accordingly the following plan was adopted:—There were two sets of needles in the galvanometer, connected with each other and the mirror; round one set was wound a coil of about 6000 ohms resistance, and round the other a coil of between 7 and 8 ohms resistance; the extremities of these coils were attached to separate terminals, and the latter coil was employed to measure the thermoelectrical effect of magnetization. The thermoelectrical current due to want of homogeneousness in the bismuth was balanced by the current from a Daniell cell sent through the other coil, which was shunted, and through a very large resistance: by altering this resistance the spot of light could easily be brought again to the middle of the scale. Some two hours were allowed to elapse, the steam during the whole of this time passing through the air-chamber, C, and the cold

water through the air-chamber, D, after which the spot of light remained steady. The solenoid, S, was now actuated by a current from six Grove cells, and a deflection ensued indicating a current *from unmagnetized to magnetized bismuth through the hot junction.* The current through the solenoid was then stopped, and the spot of light returned sensibly to its old position. The observation was repeated ten times, and then the current through the solenoid being reversed, ten other observations were made, after which the current was again reversed. The readings had to be corrected for a slight direct action of the magnetizing solenoid on the needles of the galvanometer.

The deflection due to the thermoelectrical current between magnetized and unmagnetized bismuth was very small; but, so far as could be made out, it was the same for both directions of the magnetizing current. The mean of the readings gave a deflection of 3·5 divisions of the scale; and, by independent observation with a Daniell cell, it had been ascertained that a deflection of 1 division of the scale would, under the conditions of the experiment, indicate an E.M.F. of 0·143 microvolt. Consequently the E.M.F. produced by temperatures of 100° C. and 16° C. at the two junctions of magnetized and unmagnetized bismuth would be $0·143 \times 3·5$ microvolts $= \frac{1}{2}$ microvolt. The magnetizing force was 226 C.G.S. units; so that the E.M.F. for unit magnetizing force would be ·0022 microvolt, or ·22 C.G.S. unit of E.M.F. If we divide the last number by 14×10^{-6}, the magnetic susceptibility, we shall obtain the E.M.F. which would be produced by magnetizing the bismuth to unit intensity; this is 15714 C.G.S. units. Now, according to Ewing, the E.M.F. produced in a certain specimen of soft iron wire by a magnetic intensity of 160 C.G.S. units was 6 microvolts, when the junctions of the magnetized and unmagnetized wires were at 100° C. and 16° C. respectively. Accordingly the E.M.F. produced by unit magnetic intensity would be 3·75 C.G.S. From this it is evident that, for a given intensity of magnetization, bismuth has its thermoelectrical properties altered by longitudinal magnetization 4000 times as much as iron. We see, then, that the relative changes produced by magnetization in the thermoelectrical properties of bismuth and iron are comparable with the

changes wrought in the electrical and thermal conductivities of these metals.

Grimaldi has already published researches* on the effect of both transverse and longitudinal magnetization on the thermoelectrical properties of bismuth, and finds that magnetization in either of these two directions makes the bismuth of commerce thermoelectrically negative to unmagnetized bismuth. He also finds that the thermoelectrical E.M.F. of a *pure* bismuth and copper couple is increased by both longitudinal and transverse magnetization. Now according to Ettingshausen†, who also quotes Rollmann ‡, *pure* bismuth is thermoelectrically *negative* to copper, whilst commercial bismuth is positive to copper. The following table is taken from Ettingshausen's memoir:—

TABLE III.—Bismuth alloyed with different amounts of Tin.

Potential difference, in C.G.S. units, for 1° C. between a bismuth and copper couple with one junction at 20° C. and the other at 0° C.	Number of parts, by weight, of pure bismuth in 100.
−6500	100·00
+ 280	99·05
+1950	98·54
+3910	93·86
+3390	86·90

Consequently it would seem to follow, from Grimaldi's experiments, that *both pure and commercial bismuth* are rendered by magnetization negative to the unmagnetized metal, *i. e.* the thermoelectrical current would flow from unmagnetized bismuth to magnetized bismuth through the hot junction.

Grimaldi shows that, in the following respects, the effect of magnetization on the thermoelectrical properties of bismuth

* *R. Acc. dei Lincei*, vol. iii. Also memoir, entitled *Influenza del Magnetismo sulle Proprietà Termoelettriche del Bismuto* (Palermo, 1887); *Journal de Physique*, Dec. 1887, p. 569.

† *Sitzb. der kais. Akad. der Wissensch.* 1887.

‡ Pogg. *Ann.* lxxxiii., lxxxiv., lxxxix.

resembles the effect of magnetization on the electrical conductivity:—

(1) The amounts of the two effects are comparable with each other.

(2) Transverse magnetization produces a greater effect than longitudinal magnetization.

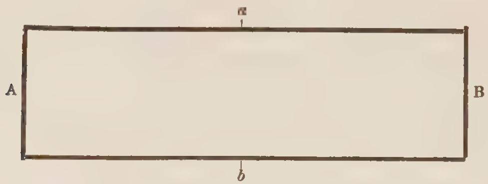
(3) Rise of temperature* diminishes the effect.

(4) The effect increases in greater proportion than the magnetizing force.

It is impossible for the author to compare his own results with those of Grimaldi; but it would seem from the above that, by using high magnetizing forces and lower temperatures at the junctions, the effect of magnetization on the thermo-electrical properties of bismuth might well be found to be some three or four hundred thousand times the effect in iron, upposing both metals to be magnetized to unit intensity.

This being the case, it is difficult to believe that the alteration due to magnetization is a real alteration of the thermo-electrical power of the metal. But, again, how are we to account for it. According to Ettingshausen†, when a plate of bismuth, A B (fig. 2), is arranged, as for experiments on

Fig. 2.



Hall's phenomenon, with its plane parallel to the flat faces of the pole-pieces of an electromagnet and perpendicular to the lines of force, whilst a current of electricity is conducted longitudinally through the plate, the excitation of the electromagnet produces a difference of temperature at the two points a and b; whilst, on the contrary, if a current of heat be conducted through the plate instead of the electrical current, there will be produced by the action of the electromagnet a difference

* At least as far as 100° C.

† *Annalen der Physik und Chemie* Band xxxi. (1887).

of potential at the two points* a and b. Further, besides the difference of potential at a and b, which is designated the transverse "thermomagnetic effect," there will be a difference of potential at A and B called the longitudinal thermomagnetic effect; and this last, says Ettingshausen, will account for the apparent effect of magnetization on the thermoelectrical properties of bismuth. Both Grimaldi† and Leduc‡, however, are of opinion that the apparent longitudinal thermomagnetic effect is produced by decrease of thermal conductivity and thermoelectrical power.

The alteration of dimensions produced by magnetization can as little account for the change in the thermoelectrical properties of metals as for the increase of resistance; for, besides the minuteness of the alteration of dimensions, in some cases the effect of loading on the thermoelectrical power is in the same direction, and in others in the opposite direction, to the effect of longitudinal magnetization, as will be seen from Table IV.

TABLE IV.

Metal.	Under longitudinal traction§.	Under longitudinal magnetization ‖.
Iron	−	−
Nickel	+	+
Cobalt	+	−
Bismuth	+¶	−

There is, however, this resemblance between the effects of magnetization on the thermoelectrical properties of iron and on its dimensions. When an iron wire is loaded beyond a certain limit, magnetization begins to produce decrease of length and increase of thermoelectrical power**.

In conclusion, when we contrast the small magnetic sus-

* Ettingshausen and Nernst, Wied. *Ann.* xxix. (1886).

† *Nuovo Cimento,* ser. 3, vol. xxii. (1887).

‡ *Comptes Rendus,* 1887.

§ A plus sign shows that the stretched metal is positive to unstretched.

‖ A plus sign shows that the magnetized metal is positive to unmagnetized.

¶ Righi, *R. Acc. dei Lincei, Transunti* (1884).

** *Cf.* Ewing, *loc. cit.*

ceptibility of bismuth with the large value of its rotational coefficient, and with the large decrease which can be produced both by transverse and longitudinal magnetization in the thermal conductivity, the electrical conductivity, and the thermoelectrical power of the metal, we must be driven to the conclusion that magnetism in all metals exerts two distinct influences; one by rotation of the molecules about their axes, the other in some way which is not yet understood. In such metals as iron, and to a less extent in cobalt and nickel, the first of these influences probably plays a not unimportant part; but in such metals as bismuth, antimony, and tellurium, the second must entirely predominate.

XXI. *Observations on the Height, Length, and Velocity of Ocean Waves.* *By* Hon. Ralph Abercromby, *F. R. Met. Soc.**

The interest in ocean waves has so much declined in recent years, that physicists have perhaps scarcely realized how much more easily measurements can be taken now than formerly.

In the old days wave-heights could only be ascertained, more or less, by estimation; while the length and speed could only be determined by a common watch. Now-a-days the aneroid can easily measure small vertical heights to within one or two feet; while the fly-back chronograph enables time to be measured to the $\frac{1}{5}$th second, without taking the eye for one moment off the object to be watched.

The following observations were taken on board the S.S. 'Tongariro,' in various parts of the S. Pacific between New Zealand and Cape Horn, in the month of June 1885.

Height was measured by a $4\frac{1}{2}$-inch aneroid with a very open scale, divided to the $\frac{1}{100}$th inch; so that the readings could be taken at a glance to 0·025 inch, or, when time allowed, to 0·020. The instrument is an extremely good and accurate barometer. The altitudes were all calculated on the simple assumption that a difference of 1 foot in height is given by a difference of 0·001 inch of pressure. Any error which could arise between this reduction and that by a more rigorous method would be far less than the other errors of observation.

* Read February 25, 1888.

So far the observation was simple enough; but the great difficulty arose when the height of the eye above the sea-level, at the moment of observation, had to be estimated. For instance, when the barometer was at its lowest point the surface of the water might be 10 feet below the eye; but when the crest of the wave rushed past, the height might be reduced to 1 or 2 feet. Here I had to trust to my own estimation by eye, aided by a few rough measurements with a piece of string down the ship's side. The principal uncertainty of the results depends on errors of this estimation. I do not think that the aneroid errors would ever be more than 2 or 2·5 feet, while those of estimation might be at least 2 feet either way.

Length and velocity were determined by standing in a suitable position with a chronograph, and measuring (1) the interval between the time when two successive crests reached the stern, and (2) the time that the crest of the first wave took to run the length of the ship. Then the length of the ship, her speed, and course relative to the direction of the wave's progress being known, the velocity and length of wave could be readily calculated by the following formulæ :—

Let t = time from crest to crest, in seconds;
T = time of first crest running the length of the ship, in seconds;
l = length of wave, in miles;
v = velocity of wave, in miles per hour;
k = speed of ship, in miles per hour;
L = length of ship, in fractions of a mile;
θ = angle between course of ship and direction of wave-progress.

Then, for a following sea,

$$v = L\cos\theta\frac{3600}{T} + k\cos\theta;$$

$$l = (v - k\cos\theta)\frac{t}{3600}, \text{ in miles,}$$

$$= (v - k\cos\theta)\frac{t}{3600} \times 5280, \text{ in feet.}$$

The calculations were made with sufficient accuracy by means of a slide-rule.

It is manifest that it is only possible to obtain an approximation to any of the desired data. Standing near the stern it is not easy to observe the precise moment when the crest reaches the bow; two successive waves rarely run in exactly the same direction; and, with a heavy following sea, the ship yaws about so much that the angle between her course and that of the waves can only be estimated approximatively.

I found great advantage in using cards for the original records, as observations must often be made in rain, sleet, spindrift, and frequently at temperatures near the freezing-point, when the fingers are apt to get benumbed.

The following are some of my most interesting results :—

June 8, 1885, lat. 47° S., long. 175° W.—Sea too irregular to measure individual wave-heights or lengths, but the barometer indicated about 12·5 feet for the vertical motion of the point of observation below decks. Then, pretty constantly, the surface of the sea was about 7 feet below the port-hole in the troughs, but only 1 foot at the crest. This would give the waves an average height of about 18·5 feet.

The velocity of the waves could be got much better. Five observations gave the accordant speeds of 29, 28, 31, 33, and 30 miles an hour, or an average of 30·2 miles.

The following data were used in the calculation :—Ship, length 380 feet; course S.E.; Speed 14 knots. Sea running to S.S.E., and therefore following.

June 10, 1885. Lat. 51° S., long. 160° W. Height.—I found it impossible single-handed to estimate the height of the water at the moment the aneroid was read, so I took a constant difference of 6 feet for the difference of the height of the eye at trough and crest. The following is an example of the readings :—

Trough.	Crest.	Difference.	Diff. height eye, trough and crest.	Estimated h ight, in feet.
28·955	28·975	·020	6	26
·97	·985	·015	6	21
·9675	·985	·0175	6	23·5
·960	·980	·020	6	26

These waves were not consecutive, and the difference between the extreme heights is a good deal larger than the height of any single undulation. The highest reading of the series (not given here) was 28·99, the lowest 28·955, which would give a difference in height of 35 feet.

The length and velocity were measured on deck just before the heights, under the following conditions :—

Ship: length 380 feet, course E.S.E., speed 14 knots. Sea running to S.E., and therefore following.

The following are some of the best observations :—

Time of crest, running length.	Time between two crests.	Velocity. Calculated.	Length. Calculated.
12·5″	19″	32 miles.	507
11·0	15	35 „	470
9·0	...	39·5 „	
16·0	17	28·5 „	358

This sea would have been logged as 6 or 7 on the ordinary scale of 0–8. The wind was blowing a moderate to hard gale from N.W. with heavy squalls, and was logged 7 on Beaufort's scale of 0–12. During some of the squalls, with thunder and lightning, the force rose to 8; and though the ship was never nearly pooped, we managed to split a topsail. I think this might be taken as a fair average sea in the S. Pacific. The waves were far too irregular to allow of any attempt being made to determine the ratio of height to length or velocity.

July 16, 1885. Lat. 55° S., long. 105° W.

Trough.	Crest.	Difference.	Diff. of height eye.	Estimated height.
29·38	29·4025	·0225	6	28·5 feet.
·38	·42	·040	6	46 „
·3775	·40	·0225	6	28·5 „
·38	·41	·030	6	36 „

The readings in the trough belonging to this series are remarkably uniform.

Length and velocity were measured from deck under the following conditions :—Ship, length 380 feet ; speed, 14 knots; course to E. ; sea from W., and therefore right aft.

Time of crest, running length.	Time between two crests.	Velocity. Calculated.	Length. Calculated.
12·5″	14·25″	35·5	445
12·5	16·5	35·5	485
8·0	16·0	47·5	765

This was the heaviest sea we encountered during the whole voyage. The wind was blowing a hard gale from S.W., with squalls of hail and sleet. The sea was certainly high, but did not appear excessively so to the eye. The ship broached to and lay in the trough of the sea for nearly half an hour while a gland in the engine-room was being packed ; but no harm was done beyond splitting a topsail. The ship rolled tremendously, but no seas swept over her.

Under these circumstances the height of 46 feet, given by one set of observations, seems excessive. The actual vertical lift of the cabin was undoubtedly 40 feet ; for, as I noticed it at the time, there is not a mistake in the records. Any error in the true height must come from the estimate of 6 feet for the difference of the height of the eye on the crest and in the trough.

It will be noticed that the relation of length and velocity to height is very irregular ; but this is due to the character of the waves, and not to errors of observation. On all the days the waves were running irregularly. We never saw crests nearly a mile long chasing one another with a well-defined trough between them ; but the seas were so confused, that sometimes after one big crest nothing followed but some small waves. There was nothing to call a cross sea ; but there were many series of waves of different lengths running pretty much in the same direction, which were constantly interfering with one another. An eye estimate of height is

always delusive, but I was surprised that the waves measured as much as they did. I have seen a heavier sea in the Atlantic; so that if we take only 40 feet as the highest of this series, it is perfectly certain that much greater heights are sometimes attained.

Fitzroy says that, even during many years spent at sea, a man only observes really high waves once or twice during a lifetime; and this view is confirmed by every sailor I have conversed with. Taking my own measurements as representing only ordinary seas, I am certain that 60 feet at least from trough to crest must be attained by exceptional waves.

It may be interesting to compare the results given here with those obtained by other observers. The following figures, except those in the last line, are taken from Dr. Krummel's *Ozeanographie*.

Table of the Maxima Dimensions of Waves.

Authority.	Locality.	Height, in feet.	Velocity, m.p.h.	Length, in feet.	Period, in seconds.
Lieut. Pâris	Indian Ocean.	......	33·6	374	7·5
Admiral Molter...	....	......	80·0	2703	23·0
Captain Ross * ...	Near C. Good Hope.	22·97	90·0	1902	
Captain Chüden..	33° S., 107° W.	33–36	......	984–1312	
D'Urville	....	98·0			
Scoresby...........	....	42·0			
'Novara'	....	36·0			
'Challenger'	....	23·0			
Abercromby	S. Pacific.	46·0	47·5	765	16·5

It is manifest, from an inspection of the above table, that the discrepancies are enormous. I cannot but think that the extreme lengths recorded by Admiral Molter and Captain Ross must have not allowed for the interference of following waves. No doubt such lengths might be observed between two notable crests, but most probably there would be some smaller undulation between. I never took the time between two crests unless they manifestly belonged to a simple wave.

The three sets of observations before noticed were taken on the only three days on which at all big waves were met with. They serve to show the difficulties of wave-measurement; but the greatest obstacle is the uncertainty of meeting first-

* Dr. Krummel's figures do not agree with those given in other places for Capt. Ross's results.

class specimens. If ever a wave-measuring party should be organized, I think it should be arranged as follows.

Three observers, A, B, and C, would be necessary. A would command the party, say when the instruments were to be observed, note personally the height of the deck from the water, and enter all the readings on a suitable card. B would have a suitable aneroid, and confine his whole attention to that instrument. C would be furnished with two chronographs.

When a crest touched the stern of the ship, A would give a signal, on which B would read his aneroid, while C started both his chronographs. A would first note the height of the deck by marks on the ship's side; and, if need be, read a simple clinometer to allow for the roll of the ship before entering the records of his assistants. When the crest reached the bow, A would give another signal for C to stop one chronograph. In the trough, B would read his aneroid, while A noted the height of the water; and, finally, as the next crest came on, B would read his aneroid, C would stop his second chronograph, while A noted the height of the water and entered all the records.

By this means, and with a careful selection of tolerably undeformed waves, I think that the measurements of undulations could be much more satisfactorily obtained than heretofore; and I only regret that the means at my disposal did not enable me to do more towards this important line of research.

Summary.

The results of this paper may be summarized as follows:—

Several sets of observations between New Zealand and Cape Horn, with an aneroid barometer and chronograph, gave for the largest waves a height of 46 feet, a length of 765 feet, a speed of 47 miles an hour, and a time-period of 16·5 seconds.

As nothing but the ordinary heavy weather of these latitudes was experienced, it is certain that waves must sometimes attain a height of at least 60 feet.

Really big seas are of very rare occurrence.

The great discrepancies in the observed elements of waves given by different observers is doubtless due to the varying lengths of every series of undulations, which therefore always make a more or less confused sea.

XXII. *Experiments on Electrolysis.*—Part I. *Change of Density of the Electrolyte at the Electrodes.* *By* W. W. Haldane Gee, *B.Sc., Assistant Lecturer in Physics, and* H. Holden, *B.Sc., Bishop Berkeley Fellow in Physics, of the Owens College, Manchester*.*

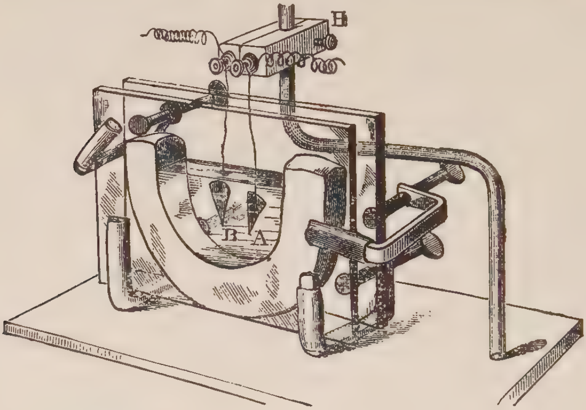
Whilst studying some electrolytic polarization phenomena with palladium electrodes in dilute pure sulphuric acid, a liquid was seen, *after a reversal of the current*, to flow downwards in streaks from the anode. Not being able to find any reference to the formation of streaks, for whose appearance the reversal of the current was necessary, it was decided to investigate their character. Further, it was thought that the occluded hydrogen might, on reversal of the current, unite with the nascent ion liberated at the anode, and thus effect chemical changes of an interesting character.

Some little care in observation and adjustment of the light is necessary in order to see the streaks, which, like the surrounding electrolyte, are colourless, and are only visible on account of the difference between their index of refraction and that of the main bulk of the electrolyte. A vessel with parallel sides of good plate-glass was employed, and a mirror used to reflect light obliquely into the cell. The arrangement which has been found to be most convenient is shown in the figure, and consists of a cell made of two pieces of plate-glass about 15 centim. square. A piece of india-rubber, 25 centim. long and 3 centim. square section, is bent in a semicircular form and clamped between the two glass plates by means of four iron screw-clamps. In this way a water-tight cell, about 1·5 centim. broad, is obtained which may readily be taken to

* Read February 25, 1888.

This is the first of a series of papers on Electrolysis and Electrolytic Polarization, descriptive of experiments made, during last year, at the Owens College Physical Laboratory. An abstract of the experiments made, to the end of August 1887, was submited to the British Association Meeting at Manchester. We desire to acknowledge the assistance received up to that time from Mr. C. H. Lees, B.Sc., Derby Mathematical Scholar of the Owens College. His cooperation has since been discontinued, owing to absence at Strasburg.

pieces for the purpose of cleaning. The electrodes have usually consisted of two pieces of palladium, about ·05 centim.



thick and with a surface of ·5 square centim., fastened to platinum wires and supported by means of a convenient electrode-holder.

Streaks obtained in dilute pure Sulphuric Acid.

The electrodes were first heated to redness in order to drive out any occluded gas, and then cleaned by means of glass-paper. They were then placed, by means of the electrode-holder H, in the electrolytic cell containing dilute pure sulphuric acid. The current was sent from one electrode (A) to the other (B), which is thus the kathode, for a certain time and then reversed. On reversal no gas appears at first from B, which is now the anode, but streaks resembling a dense liquid are with careful observation seen flowing downwards from this electrode. After a time, depending on the size of the electrodes, the strength of the current, and its duration in the first direction, evolution of gas at B begins, and the streaks simultaneously cease to be visible. The streaks are to be seen streaming from one electrode only at a time, and always descend from that palladium electrode which first serves as kathode and then as anode.

When platinum electrodes are substituted for the palladium ones, we have not been successful in obtaining the streaks; it

is, however, only necessary that one of the electrodes should be palladium in order to obtain the streaks.

An experiment was made in order to ascertain the part which the reversal of the current plays in the formation of the streaks. The current was sent in one direction through the electrolyte, the palladium electrodes then taken out, washed, and their surface cleaned with glass-paper. On now reversing the current and replacing the electrodes, the streaks are quite as evident as if the electrodes had not been taken out. But if, on taking the electrodes out, they are heated to redness instead of merely having their surface cleaned, on replacing them and reversing the current the streaks are not seen, and gas appears immediately from the anode. We concluded from this experiment that the function of the current in the first direction was to fill with hydrogen that electrode at which the streaks appear in the second direction of the current; and that the reason why, after a time, the streaks ceased to be visible, and simultaneously gas began to be evolved at the anode, was that by this time all the occluded hydrogen had been used up. It was therefore thought probable that the streaks were formed by a combination of the occluded hydrogen with the ion (either SO_4 or some of its components, such as oxygen) which is liberated at the anode in the second direction of the current. The theory that the streaks were composed of concentrated sulphuric acid seemed open to great objections; for we know that concentration takes place at the anode in dilute sulphuric acid without a reversal of the current, and also does not depend on the use of palladium electrodes; but the streaks are, as is shown above, only to be obtained after a reversal of the current, and are not to be obtained with platinum electrodes. It was therefore decided to test, as far as practicable, other possible combinations, such as hydroxyl (the streaks cannot be composed of water, for they are denser than the surrounding electrolyte), a salt of palladium, &c.

Testing for Hydroxyl.

The palladium electrodes used were rectangular strips, about 2 centim. long and ·3 centim. broad, placed vertically in 10-per-cent. sulphuric acid. For the purpose of collecting the

streaks a capillary glass tube had one end blown out into the shape of a small funnel, and was bent so that it could be lowered into the electrolyte with the finger pressed on the upper end, which projected from the cell, until the funnel part was immediately below the electrode. After a reversal of the current the funnel part was placed in this way under the anode, from which the streaks were flowing. The finger was then removed from the top end of the tube, and the funnel acted as a reservoir for the denser liquid flowing from the anode. As soon as gas appeared at the anode the finger was replaced on the top end of the tube, the latter being then removed and its contents emptied into a test-tube. This proceeding was repeated many times (at least fifty), and the total liquid collected was tested for hydroxyl by the addition of a few drops of a solution of titanic acid in strong sulphuric acid. No trace of a coloration could be perceived, and thus the presence of hydroxyl was not proved *.

* The test, which is a delicate one, has been greatly employed by Richarz in his experiments on the mode of appearance of hydroxyl at the anode in sulphuric acid (Wied. *Ann.* xxxi. p. 912, 1887). He concludes that the appearance of hydroxyl at the anode is caused by the purely chemical decomposition of the "Ueberschwefelsäure" (Berthelot's $S_2O_7 + H_2O$), which is first formed there. Traube, *Berichte der deut. chem. Gesell.* xviii. p. 3348 (1888) gives the equation for this decomposition as

$$S_2O_6(O_2)H_2 + 2H_2O = 2H_2SO_4 + H_2(O_2).$$

Richarz confirms Berthelot's statement that no hydroxyl is formed at the platinum anode unless the strength of the acid is above 60 per cent. As we obtained the streaks in 10-per-cent. acid, the above fact lends additional strength to the view that the streaks are not hydroxyl. They might consist of $H_2S_2O_8$ (Ueberschwefelsäure), which Berthelot has shown to be a minor secondary product formed during the electrolysis of sulphuric acid, whether dilute or strong (see *Ann. de Chim. et de Phys.* xiv. 1878, and xxi. 1880). The quantity so formed at a platinum anode is small, and would apparently be still smaller at a palladium anode charged with hydrogen if the following reasoning be accepted. Let us first consider the case of an anode uncharged with hydrogen. Assuming that the ion liberated at the anode is SO_4, the simplest hypothesis to account for the secondary reactions is that part of the SO_4 combines directly with the H_2SO_4 of the electrolyte to form $H_2S_2O_8$, whilst the remainder of the SO_4 (probably by far the greater part) acts on the H_2O of the electrolyte, forming H_2SO_4 and liberating O. Should, however, the anode be charged with H, the SO_4 would have an additional tendency to combine directly with the H, forming H_2SO_4. This tendency would probably greatly

Testing for Palladium Salts.

A supply of the liquid having been obtained by a similar process to the above, a solution of potassium iodide was added to it, but no evidence of the presence of palladium was obtained.

Streaks obtained in Phosphoric Acid.

Similar streaks were obtained in solutions of pure orthophosphoric acid by adopting exactly the same procedure as with sulphuric acid.

Streaks obtained in Caustic-Soda Solution.

In this case also streaks were obtained descending from one of the electrodes, but the conditions necessary for their appearance are very different to those in the case of sulphuric and phosphoric acids. *No previous reversal of the current is necessary,* and the streaks descend from the kathode immediately the current is passed. The gas (hydrogen) formed at the kathode is at first absorbed, and the streaks are seen to descend as long as the absorption takes place; but, when gas begins to be evolved briskly, the streaks disappear. By very careful observation it is seen that the liquid of which the streaks are composed is still formed, but is carried up in the current of gas to the surface of the electrolyte, from which it rebounds, giving the electrolyte round the kathode the peculiar wavy appearance which accompanies the incomplete mixing of two liquids of different densities, such as two strengths of the same solution.

Explanation of the Caustic-Soda Streaks.

This experiment led us to infer that the streaks in the caustic-soda solution are composed of concentrated alkali, which is known to be formed at the kathode, and that the

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exceed the other two, so that if sufficient occluded H be present, it would be expected that neither O would be liberated, nor $H_2S_2O_8$ formed. It is known that there is no evolution of O under these conditions; and it would be interesting to estimate the relative amounts of $H_2S_2O_8$ produced at a palladium anode when charged and when uncharged with H. The $H_2S_2O_8$ could be quantitatively determined by the method used by Richarz (*loc. cit.* p. 917).

absence of evolution of gas from that electrode is a necessary condition for their appearance and steady downward flow. The part played by the palladium in the formation of the streaks is, according to this theory, that of absorbing the hydrogen, which otherwise, in escaping from the electrode, would carry the concentrated solution along with it to the surface and thus prevent the formation of the streaks.

Explanation of Streaks in general.

Since in acids concentration occurs at the anode, the descending streaks, if seen at all, should be seen at that electrode. But taking sulphuric acid as an instance, oxygen is given off at the anode, and the palladium cannot retain it sufficiently to stop its evolution, and thus the concentrated acid will be carried up by the current of oxygen and escape observation. But if the electrode has been filled with hydrogen by previously serving as kathode, the evolution of oxygen on reversal of the current is prevented either by its direct union with the occluded hydrogen, or by the union of the ion SO_4 with the hydrogen. No gas being evolved, the concentrated acid is able to flow downwards in streaks. This explanation is borne out by the experiments with sulphuric acid detailed above, in which it was seen that, on placing freshly-heated palladium electrodes in sulphuric acid, gas is immediately evolved at the anode; but if that electrode has been previously filled with hydrogen, the oxygen, on reversal of the current, does not appear for some time at the anode, and the streaks are visible during this period.

Concentration of the solution at one electrode is accompanied by a weakening of the solution at the other electrode. Therefore, as in acids the weakening takes place at the kathode (at which the gas is absorbed), we should expect to see streaks ascending from that electrode without a previous reversal of the current. On trying the experiment with palladium electrodes placed *horizontally* in dilute sulphuric acid, this supposition is found to be warranted. The reason why the streaks ascending from the kathode were not seen in our earlier experiments is, that the electrodes had been placed vertically with their top edges a little below the surface of the electrolyte, and so the weakened solution in

ascending had naturally clung to the surface of the electrode and thus escaped detection.

The late Professor Christiani (in a work* to which we shall refer more fully in a subsequent paper) gives three instances in which he observes streaks from the electrodes, but does not offer any explanation of their mode of formation. He cites the cases of zinc electrodes in concentrated zinc sulphate and copper electrodes in concentrated copper sulphate†. In these cases, without a previous reversal, he observed streaks descending from the anode and ascending from the kathode at the same time. These results are evidently in accordance with the theory proposed above: no gas is given off at either electrode, and so the concentrated solution is allowed to descend in streaks from the anode and ascend in streaks from the (horizontal) kathode.

Another confirmation is given by the behaviour of potassium sulphate during electrolysis. After a current has been passed in the same direction through potassium sulphate, it is found, on testing with litmus paper or by cautiously adding litmus solution, that the bottom strata of liquid have become alkaline and the top strata acid, the solution being originally neutral. It follows from this that the alkaline solution produced at the kathode is denser than the electrolyte, which itself is denser than the acid solution produced at the anode. Therefore, according to theory, the streaks, if seen at all, should descend from the kathode and ascend from the anode. This is fully borne out by experiment. Starting with freshly-heated palladium electrodes, on putting on the current we immediately see the streaks descending from the kathode; and after a reversal of the current other streaks may simultaneously be detected rising from the anode.

The best conditions for seeing the streaks seem to be:—

(1) a horizontal electrode of small surface‡,

* "Ueber irreciproke Leitung electrischer Ströme," 1876, R. Friedländer; Wied. *Elect.* ii. p. 727. † Christiani, *loc. cit.* p. 100.

‡ A vertical electrode may be used with advantage if it is well below the surface of the electrolyte and if it is suspended by a horizontal wire, so that the view of the electrolyte above and below it is not interrupted. An angular electrode is very convenient, as the liquid flows most readily from points.

(2) a strong current,

(3) no evolution of gas.

The conditions (2) and (3) are incompatible with the use of platinum electrodes in dilute sulphuric acid : it is quite possible, however, that even in this case matters might be so adjusted that some trace of the formation of the streaks could be detected. In the case of platinum electrodes in solutions of sulphates of metals which do not act readily on water, such as zinc, copper, iron, &c., the metal will be deposited and no gas evolved at the kathode, and thus streaks of the weakened solution can be seen at that electrode*.

A direct proof that the streaks were formed of concentrated acid might be thought possible, in the case of sulphuric acid, on consideration of the following circumstances :—

(1) When strong sulphuric acid is dropped into dilute sulphuric acid, the strong acid sinks to the bottom of the containing vessel and remains undiffused for a considerable period.

(2) If collecting-vessels are placed under the electrodes and an arrangement fitted up by which the current may be periodically reversed, say every minute, the streaks will be formed at the electrodes and will stream into the collecting-vessels.

(3) If we now titrate with standard caustic-soda solution equal volumes of the liquid in the collecting-vessels and of the main bulk of the electrolyte, we ought to find that the former is the more acid.

It may, however, be noticed that, assuming the streaks were not concentrated acid, we should collect by this arrangement, not only the streaks, but also some of the concentrated acid which is formed at the anode. Thus, even if the acid in the collecting-vessels did become stronger, it would scarcely prove that the streaks were the cause of this†.

From the experiments detailed above, though direct proofs are apparently not available, we may draw the following general

* Christiani noted the streaks with platinum electrodes in ferrous sulphate.

† Some observations have been made by this method, but the results have not been very definite. We have designed an automatic commutator which will reverse the current at intervals of 15, 30, or 60 seconds. This arrangement will enable us to collect more easily the substance forming the streaks.

conclusions:—That when gas is not evolved at an electrode, streaks are formed there. These are due either to a concentration or weakening of the electrolyte, as in the case of solutions of acids and alkalies and some salts (such as zinc sulphate); or, in the case of other salts (such as potassium sulphate), to a chemical change in the electrolyte, yielding at one pole alkali and at the other pole acid, producing solutions of different density to the electrolyte. When gas is not evolved at an electrode these changes are still produced, but their effect in producing streaks is destroyed by the evolution of the gas.

In our next paper we hope to describe some experiments in which these effects become of great importance in changing the resistance of the electrolyte.

XXIII. *On a Method of Determining the Difference between the Phase of two Harmonic Currents of Electricity having the same Period.* *By* Thomas H. Blakesley, *M.A.**

It has been brought to my notice by both English and Foreign journals connected with science that a method of determining the difference in phase of two Harmonic Currents of Electricity having the same period, which I invented and published some years ago, forms an important part of the subject matter of a paper communicated to the Royal Academy of Sciences of Turin, second series, vol. xxxviii., by Signor Galileo Ferraris, in which that philosopher lays claim to the invention above mentioned, producing it as original, with no sign of acknowledgment that the method has before been made public.

The method consists in employing the two coils of an electric dynamometer in a peculiar way. When an harmonic current is sent through the coils of such a dynamometer in series, its reading will measure the quantity $\frac{I^2}{2}$, where I is the maximum value of such a current. In this way we may

* Read March 10, 1888.

successively determine $\frac{I_1^2}{2}$ and $\frac{I_2^2}{2}$, where the subscripts refer to two currents having the same period.

But if we place one of the coils in one circuit and the other in the second circuit, the reading of the instrument will measure $\frac{I_1 I_2}{2} \cos \theta$, where θ is the angle representing the phase-difference of the two currents, to which the name "*décalage*" has, I have no doubt with great propriety, been accorded by M. Hospitalier.

It is clear that from the three readings we can deduce the angle of phase-difference. Thus, $\alpha_1, \alpha_2, \alpha_3$ being the three readings, we have:—

$$\alpha_1 \propto \frac{I_1^2}{2},$$

$$\alpha_2 \propto \frac{I_2^2}{2},$$

$$\alpha_3 \propto \frac{I_1 I_2}{2} \cos \theta ;$$

$$\therefore \cos^2 \theta = \frac{\alpha_3^2}{\alpha_1 \alpha_2}.$$

Here I have supposed that the same instrument is used successively; and it is hardly necessary to point out that if we have three exactly similar dynamometers, or if we have three dynamometers of which the relative values of the constants are known, we can deduce θ by means of three simultaneous observations.

The real importance of the determination of the *décalage* rests in the means it affords us of determining the causes of its existence; among which I may here mention Coefficients of Induction (self or mutual), Capacity of Condensers, and hysteresis or the waste of energy involved in the reversal of electromagnetic momentum, all of which present useful fields for inquiry capable of investigation by means of this application of the dynamometer, a few of which I have myself traversed.

I wish to point out that this method and its importance (not yet fully appreciated) were published by me in the 'Electrician' newspaper of October 2, 1885; and the entire

series of papers, of which this formed one, was republished in book form at the end of the same year. This was before I had the honour of membership in the Physical Society, or doubtless I should have then brought the subject more immediately under your notice; but upon my being elected a member I lost no time in presenting a copy to this Society, and those who care to examine this question will find therein a chapter devoted to this very employment of the dynamometer to determine Coefficients of Self- and Mutual-Induction and the Capacity of Condensers.

March 1888.

XXIV. *On the Numerical Relation between the Index of Refraction and the Wave-length within a Refractive Medium, and on the Limit of Refraction.* By T. Pelham Dale, *M.A.**

The following equation (see Sir Geo. Airy's Treatise on the Undulatory Theory, ed. 1877, p. 93) is proposed as expressing the relation between wave-length and velocity of propagation in an isotropic medium:—

$$v=\sqrt{\left(1-\frac{1}{2^{\frac{3}{2}}}\right)\frac{m}{}}\cdot\frac{\sin\frac{\pi h}{\lambda}}{\frac{\pi h}{\lambda}}\quad .\quad .\quad .\quad .\quad .\quad .\quad (\alpha)$$

v is here the velocity of propagation, h the distance between the undulating particles, λ the wave-length, m the absolute force of attraction, supposed to follow the law of the inverse square.

For our purpose this may be put under the form

$$v=k\sin\frac{\pi h}{l}\Big/\frac{\pi h}{l}\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad (\beta)$$

It is convenient to reserve λ to signify the wave-length in free æther, and to use l for the wave-length within the medium. The values of the wave-lengths in free æther are those given in the text-book of Glazebrook and the tables of Lupton.

Now, knowing the index of refraction for any fixed line by observation of the index μ, the corresponding value of l is given by the equation

$$l=\frac{\lambda}{\mu}.$$

* Read February 11, 1888.

A convenient notation is l_A, λ_A, μ_A, &c., for index and wave-lengths of the line A &c.

Now if V be the velocity of propagation in free æther,

$$v=\frac{V}{\mu},$$

whence from (β) we have

$$\frac{\pi h v}{l}=k \sin \frac{\pi h}{l}.$$

Calling $\frac{\pi h}{l}$, θ, we have

$$\frac{\pi h v}{l}=k \sin \theta ;$$

for any other value l_1 corresponding to index μ_1 we have

$$\frac{\pi h v}{l_1}=k \sin \theta_1,$$

and

$$\frac{\pi h}{l_1}=\theta_1 ;$$

or

$$\theta_1=\frac{l}{l_1}\theta. \quad \dots \dots \dots \quad (\gamma)$$

Hence if we can find a value of θ for any one wave-length and index, all the others can be found from the observed wave-lengths and corresponding indices.

The angle θ can be readily found, if we know two indices, one corresponding to any given wave-length, and the other corresponding to double the wave-length, both *within* the medium.

For
$$\frac{\pi h}{l}=\theta,$$

then
$$\frac{\pi h}{2l}=\frac{\theta}{2},$$

$$\frac{\pi h v}{l}=k \sin \theta=2k \sin \frac{\theta}{2} \cos \frac{\theta}{2},$$

$$\frac{2 \pi h v'}{2l}=2k \sin \frac{\theta}{2} ;$$

$$\therefore \frac{v}{v'}=\cos \frac{\theta}{2},$$

or

$$\frac{\mu_{2l}}{\mu_l}=\cos \frac{\theta}{2}. \quad \dots \dots \dots \quad (\delta)$$

Now μ and μ_1 can be found either by observation or interpolation. If H be the line chosen, then, practically, the index of a wave-length twice as long in the medium will not be far from A; this we may call μ_{2H} and speak of it as the index of the octave below.

It is evident that if k remain constant, $\frac{k \sin \theta}{\theta}$ continually approaches k as θ diminishes. Now if l be increased and h and m remain constant, θ diminishes; hence there must be a limit of refraction in the case of waves of very great length. This limit, denoted by ν, is evidently found by the equation

$$\nu = \frac{\mu \sin \theta}{\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\epsilon)$$

This is in effect the limit found by Cauchy in a somewhat modified form.

Since

$$\nu = \mu \frac{\sin \theta}{\theta},$$

$$= \mu_1 \frac{\sin \theta'}{\theta'},$$

it follows on substitution of $\frac{\pi h}{l}$ for θ and $\frac{\lambda}{\mu}$ for l, and dividing out, that

$$\sin \theta = \frac{\lambda}{\lambda_1} \sin \theta_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\zeta)$$

Combining this with the equation above,

$$\theta = \frac{l_1}{l} \theta_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\eta)$$

we can find a value of θ which satisfies both these equations from any two values of wave-length and corresponding index. And from this value of θ the corresponding values of θ to any other index and wave-length can be found.

In practice this equation can be solved by trial without much trouble, especially when a large number of indices of substances of similar refractive power have to be investigated, when a table may be made once for all, and from this θ may be found by inspection.

The wave-lengths made use of in this paper are generally those corresponding to lines A and H. These are both within the visible spectrum, yet far enough apart to render errors of

observation of little consequence. In the case of the liquids examined I am altogether indebted to Dr. Gladstone, who furnished me with those contained in Table III. for the purposes of this investigation. When the line B is given, the octave ray to H may be obtained from A by interpolation, and these values were calculated by this method. It gives results sufficiently near the truth, but has been verified in case of CS_2 (temp. 1°·5) and mint hydrocarbon by reference to the two indices A and H, so that the table depends on these two indices only. It will be seen that in the cases examined $\frac{\nu-1}{d}$ is constant, within the limits of observation, unaffected by temperature.

Dr. Gladstone and myself, in our joint paper, read before the Royal Society, May 6, 1863, and printed, p. 317, in the Transactions, have pointed out that in consequence of an accidental relation amongst the coefficients used in Cauchy's series, that the equation

$$\nu=\mu_{H}-3\ (\mu_{F}-\mu_{B})$$

holds with considerable accuracy in the cases examined. It appears generally to give a fair approximation to the limit*, and thus to interpolate indices within the visible spectrum.

While, however, this investigation was in progress Prof. Langley's first paper on the determination of wave-lengths in the invisible spectrum appeared (Phil. Mag. March 1884). He there gives the wave-lengths of lower heat-rays down to length 2·030, and shows that the observed indices of refraction are not only below the limit obtained by Cauchy's formula, but even below those found by other formulæ. The material of his prism was flint glass. I found that the values derived from θ_{H} and θ_{2H}, while fairly accurate in the visible spectrum, were increasingly discordant in the longer wave-lengths. If θ_{H} were increased the lower indices were found more accordant; but then the visible spectrum was not adequately represented within the limits of any error which could be imagined possible. This seemed to throw a doubt on the existence of a limiting value. It appeared, however, that the glass prism absorbed the longer waves considerably. The

* The divergence in case of CS_2 is, however, considerable, and ν is too large in comparison with that given by the above equations. Especially does this occur in highly dispersive media.

Phil. Mag. for May 1886 contained another paper by Prof. Langley, the material of the prism being rock-salt. On comparing the values found by observation with those furnished by the above equations, the agreement was sufficiently close to encourage investigation, and, accordingly, the calculations were carried further, the approximation being to single seconds in the value of θ and every value found.

The results are embodied in Tables I. and II. It will be seen in the case of rock-salt that the agreement between the values obtained from the sine—which involve only wave-lengths in free æther—and those obtained from the arc, which involve the two indices, and which should be equal to the former, are very close. The errors, it will be observed, are of small amount and not of uniform sign, possibly, however, showing a small tendency to increase on the side of angle from the sine in the case of the lower indices. Turning to the table of indices in the case of flint glass, we see that the angle for H, 24° 18′ 40″, gives fairly accordant values down to wave-length 0·940, and not far from the truth to 1·270; after this the values increase in the case of the sine over that of the arc, or, in other words, the calculated index comes out too large. This is the more important because the proportionate increase on so small an angle as 4° makes a large increase in the resulting index. If the value of θ_H be increased, this difference is diminished in the lower values; but it would require an increase far greater than could possibly be allowed as due to errors of observation in order to include the lower values, and the upper would become greatly discordant with observation.

It was, however, noticed by Prof. Langley that the flint-glass prism appeared strongly absorbent of the lower rays. It would no doubt be unsafe to reason very confidently from results obtained from only two substances. We may remark, however, that the approach to limit of refraction, if it exist, must display itself in observation by a small increase in the index for a large proportionate increase in the wave-length. Any disturbing cause then must become, proportionately, also increasingly effective as the wave-length increases. Hence must arise a tendency to mask the limiting value if attempted to be found by observation. Now in the case of

flint glass we can hardly suppose but that some effect is produced by the change of temperature which results from the passage of the heat-rays through the medium. In the case of fluids the heat lowers the index, and thus the limit would be lowered also, and in somewhat greater proportion. It is, in the absence of sufficient data, not easy to say what would be the effect of this change of temperature in solids. In the case of glass, the *course* of the ray through the medium would possibly be raised in temperature above the surrounding mass; and thus there would be a cylinder of glass through which the pencil of rays passed, which might be in a state of constraint as compared with that around it. That a state of constraint exists is shown by the glass "flying" when suddenly heated.

However this may be, it seemed desirable in the first place to continue the investigation with the data at hand, and to calculate the value of the limit as given by the above equations, using as data two observed indices only. The results are given in the tables, and will be seen to include a considerable number of fluids differing widely in specific gravity, optical properties, and chemical constitution. The list it will be noticed includes certain isomeric bodies and also several differing by a single component.

The first fluid examined was bisulphide of carbon at different temperatures, the data for which were furnished by Dr. Gladstone to me for the purpose of this investigation. It will be seen that the quantity $\frac{\nu-1}{d}$ is practically a constant. The same result is observable in the case of mint hydrocarbon and benzene. The range of temperature is not indeed very large, but within the limits taken it is evident that the specific gravity and the quantity $\nu-1$ increase or diminish *pari passu*, and there is but little doubt that this relationship obtains generally. It would be very desirable indeed to have longer ranges of temperature* if these could be procured without corresponding errors which high temperature would almost certainly have a tendency to introduce.

* This is especially desirable near boiling and freezing points. It is possible that the specific gravity at boiling-point might reveal relations at present masked by the arbitrary character of specific gravities taken at any temperature convenient for observation.

It is of interest evidently, in the case of a highly dispersive substance, to determine how far the different indices yield the same limit, as they clearly should do if the formulæ are exact. For the determination of this a value of θ was chosen which should yield the value which would satisfy θ_A; and then wave-lengths of lines B C D E F and G in free æther and in the medium were treated so as to find θ_B &c. If the sine and arc give the same result, then calculation and observation agree. It will be seen that in the case of CS_2 there is an outstanding difference which amounts to not quite three minutes of arc, and increases from A to H. This is equivalent to saying that the formulæ give correct results to the third place in the case of CS_2. This is not so good as the rock-salt in Table II. The index of G is most affected. The difference was noticed by the late Professor Baden Powell, in his 'Undulatory Theory as applied to the Dispersion of Light,' London, 1841. This matter deserves further investigation, but it is a matter also which will involve a considerable amount of calculation; thus it seemed right to postpone this for the present, as being better policy to obtain first a sufficient number of values of ν in case of media varying considerably in optical properties.

It seemed also desirable to include in the choice of bodies several which were isomeric. There is a certain analogy in these to a body at different temperatures, as in either case we are dealing with the same chemical elements. The result is, as will be seen from examining the tables, that in many cases $\frac{\nu-1}{d}$ is very nearly a constant. In no case yet examined is the divergence very great. Thus in benzene and styrolene, cresol, metacresol, and benzil alcohol, benzil chloride and chlorotoluine, methyl citraconate and methyl mesaconate, the agreement is close; but in case of picoline and aniline, as also acetone—with which butyric ether agrees—as compared with allyl alcohol there is a difference of about three units in the second place *. It may be merely an accidental coincidence, but it appears that the quantity $\frac{\sqrt{h}}{d}$ in these two last-mentioned cases is nearly the same, but in

* But in these latter the specific gravity is evidently too high in the case of acetone, and in a less degree with butyric ether.

others the divergence is found to be greater. This also requires further investigation.

It must, however, be remembered that in all these the limit is derived solely from A and H. It is evident, then, that if either of these indices for A or H are in error, or *are affected by anomalous dispersion*, all the rest will be affected. Yet, considering the nature of the investigation,—which is, given certain indices to find the rest—it is evident that the fewer which are assumed as data the more confidence we may have, if we find that the rest are all calculable within reasonable limits. This, then, is the result of the present investigation: given two indices, all the rest can be found within limits, which are in a considerable number of instances very fair approximations to the truth. As, however, in the cases observed the outstanding differences are in the same direction, and increase apparently towards the more refrangible end of the spectrum, we have an indication of law, which may be sought for both in the circumstances of the experiment and in the mathematics of the problem.

It will be observed that the effect of substitution on the quantity $\nu-1$ is very marked indeed. This is shown in the case of ethyl sulphide and ethyl thiocarbamide, and still more in that of the substitution of iodine and bromine for chlorine. Iodine is peculiarly effective in increasing the specific gravity in far greater proportion than it raises the value of the refractive and dispersive powers. No simple relation between either density and $\nu-1$ or the quantity denoted by h is as yet apparent. I have tried the square and cube, and the square root and cube root of h, as well as the simple power. I have noticed above that $\frac{\sqrt{h}}{d}$ is in case of some isomeric bodies nearly a constant, but this may be a mere coincidence. It is, however, worth noticing, as $\sqrt{h}$ enters the coefficient k of Airy's equation cited above. When the logarithms are at hand, the calculations are so simple that even in the absence of any theoretical considerations it seemed worth while to try them.

It will be observed that the equation, by (ζ) and (η),

$$a \sin \theta = \sin m\theta$$

is impossible if $a \sin \theta$ is greater than unity. Now as a is the ratio of two wave-lengths in free æther, denoted in this investigation by $\lambda : \lambda_1$, we may ask what happens if the ratio of λ_1 the upper wave-length to λ the lower exceeds the ratio unity. Now in all cases yet examined, with the exception of selenium, this shorter wave-length is beyond violet and indeed the visible spectrum. In selenium it must lie somewhere near E; in solid phosphorus and sulphur it is beyond the furthest limit of the spectrum towards its more refrangible end, given by Prof. Langley, *i. e.* 0·3727. The results are given for these three substances in Table IV. annexed. It will be seen that phosphorus reaches the limit at about 2684 and sulphur at nearly 2664. We may observe as a coincidence that the behaviour of selenium in relation to light is remarkable, while its red colour shows that the violet end of the spectrum is absorbed. The results in Table V. are supplemental, calculated to the nearest minute, from determinations recently supplied by Dr. Gladstone, of ν for water and alcohol.

Abstract of Results.

1. That the relationship between the limit of refraction (ν) found by the equation $a \sin \theta = \sin m\theta$, where a is the ratio of two wave-lengths in free æther and m the corresponding ratio in the medium, give a value which has the property expressed by the equation $\frac{\nu - 1}{d}$ a constant, where d is the density of the medium and ν its limit of refraction at the same temperature.

2. That in isomeric bodies the same relation obtains closely in a large number of the bodies examined.

3. In all the cases examined ν is a quantity not *very* far below μ_A.

4. That the substitution of one chemical element for another has a very great influence on the quantity $\frac{-1\nu}{d}$, and substances with high chemical equivalent have, as far as examined, a far greater influence in increasing the density than in raising the limit.

5. That the equation $a \sin \theta = \sin m\theta$ gives in the cases

examined an approximate value for all the other indices, two being assumed as data. In the case of rock-salt this holds good far down in the invisible heat-spectrum, and also in the ultra-violet, but fails somewhat at a certain point in the flint-glass spectrum, and also, to a less degree, at the upper end of the visible spectrum of bisulphide of carbon.

It is evident that for a certain value of a and θ the equation $a \sin\theta = \sin m\theta$ becomes impossible. This would mean that there is an upper limit which is given by a certain value of wave-length, beyond which the formula gives impossible results. In the case of selenium only, of the substances examined, is this limit within the visible spectrum. The analogy with the critical angle of total reflexion gives this circumstance considerable interest.

It is evident, then, in the present state of our data, that in all cases examined the formula of Sir Geo. Airy gives an approximate value of an index by means of two others, and in this way they can be calculated; but that, as a rule, the lower indices come out too high as compared with the upper. A cause for this may perhaps be suggested in the heating of the medium due to the absorption of the rays; and as there seems to be evidence of absorption of short waves above the longer ones, given by recent stellar photometric determinations, this absorption would perhaps, if included in the conditions of the problem, account for some of the outstanding difference. This and anomalous dispersion —itself apparently depending on absorption—opens an interesting field of inquiry, and in this direction I propose to pursue the investigation.

Since the above was written I have been informed by Dr. Gladstone that he in some cases used a screen of solution of alum, but found that practically nothing was gained by its use. This fact, then, must be taken into account in any attempt to explain the outstanding differences between theory and observation.

Table I.—Langley's Flint-glass Prism.

Line.	Wave-length.	Index.	θ from sin.	θ from arc.	θ from sin.	θ from arc.
			° ′ ″	° ′ ″	° ′ ″	° ′ ″
O......	0·34400	1·6266	28 21 6	28 22 50	29 16 30	29 17 10
H ...	0·39679	1·6070	**24 18 40**	do.	**25 5 0**	
F......	0·48606	1·5899	19 38 10	19 38 7	20 14 50	20 15 33
D......	0·58890	1·5798	16 6 20	16 6 26		
C......	0·65618	1·5757	14 24 57	14 24 57	14 51 15	14 52 25
A......	0·76009	1·5714	12 24 20	12 24 20		
	0·815	1·5697	11 33 46	11 33 42		
	0·850	1·5687	11 4 50	11 4 43		
	0·890	1·5678	10 34 30	10 34 28		
	0·910	1·5674	10 20 30	10 20 23		
	0·940	1·5668	10 0 21	10 0 21		
	1·130	1·5636	8 18 43	8 18 23		
	1·270	1·5616	7 23 19	7 22 53		
	1·360	1·5604	6 53 56	6 53 15		
	1·540	1·5576	6 9 30	6 8 31	6 20 30	6 20 13
	1·580	1·5572	5 56 0	5 54 20		
	1·810	1·5544	5 10 40	5 9 10	5 19 50	5 19 7
	1·870	1·5535	5 0 40	4 59 10		
	1·980	1·5520	4 43 55	4 42 20		
	2·030	1·5515	4 37 11	4 35 17	4 44 32	4 41 1

Table II.—Rock-salt Prism.
(Prof. Langley's Determinations, Phil. Mag. May 1886, no. 132.)

Line.	Wave-length.	Index.	θ from sine.	θ from arc.	Difference.	Limit of Refract. Sine.	Limit of Refract. Arc.
			° ′ ″	° ′ ″	′ ″		
M ...	0·3727	1·57486	24 46 35	24 46 45	+0 10		
L......	0·3820	1·57207	24 8 5	24 8 19	0 14		
H_2 ...	0·3933	1·56920	23 23 50	23 24 8	0 18		
H_1 ...	0·3968	1·56833	**23 10 49**	datum.		1·52570	
G ...	0·4303	1·56133	21 17 0	21 17 7	0 7		
F......	0·4861	1·55323	18 44 47	18 44 47	0 0		
b_4 ...	0·5167	1·54991	17 35 41	17 35 40	−0 1		
b_1 ...	0·5183	1·54975	17 32 19	17 32 16	−0 3		
D_1 ...	0·5889	1·54418	15 22 49	15 22 49	0		
D_2 ...	0·5895	1·54414	15 21 50	15 21 52	+0 2		
C......	0·6562	1·54051	13 46 12	13 46 12	0		
B......	0·6867	1·53919	12 8 49	12 8 50	+0 1		
A......	0·7601	1·53670	11 51 12	11 51 13	+0 1		
ρ......	0·94	1·5328	9 33 53	9 33 58	+0 5		
φ......	1·13	1·5305	7 56 42	7 56 40	−0 2		
ψ......	1·38	1·5287	6 29 49	6 29 51	+0 2		
Ω......	1·82	1·5268	4 55 24	4 55 14	−0 10	1·5251	1·5249

H_1 is found from indices of H_1 and A, and from H all the other lines are deduced. Molecular distance $h=0·032570$; $\mu_H-3(\mu_F-\mu_B)=1·52621$.

Table III.

Bisulphide of Carbon.

Temp. t.	Sp. Grav. d.	Index A. μ_A.	Index H. μ_H.	Angle A. θ_H.	Limit. ν.	Mol. dist. h.	Specific Limit. $\frac{\nu-1}{d}$.
1°·5	1·2909	1·6227	1·7159	18° 38′ 2″	1·5944	·048505	46048
23·0	1·2594	1·6070	1·6972	18 25 50	1·5799	·04844	46046
38·0	1·2494	1·6026	1·6922	18 25 30	1·5752	·048559	46038
Another Specimen.							
10·0	1·2793	1·6153	1·7078	18 37 43	1·5869	·048673	45885
24·5	1·2593	1·6045	1·6954	18 30 45	1·5770	·048616	45819

Comparison of Values of Limit calculated from sine θ_H and arc θ_H with corresponding angles for CS_2 at 10°.—$\theta_H = 37° 44' 40''$.

	Line A.	B.	C.	D.	E.	F.	G.
Sine ...	18° 37′ 43″	20° 42′ 50″	......	24° 19′ 59″	......	29° 54′ 4″	34° 19′ 40″
Limit...	1·5870	......	......	1·5862	......	1·5843	1·5847
Arc ...	18 39 46	20 41 52	......	24 18 52	......	29 55 58	34 16 52
Limit...	1·5869	......	......	1·5861	......	1·5849	1·5848
Another value obtained by interpolation, $\theta_H = 37° 29' 20''$.							
Sine ...	18° 35′ 20″	20° 40′ 10″	21° 40′ 40″	24° 16′ 50″	27° 23′ 30″	29° 54′ 50″	34° 13′ 40″
Arc ...	18 35 10	20 40 40	21 40 0	24 15 30	27 21 10	29 51 50	34 11 10

The value of ν lies between 1·5867 and 1·5852.

Mint Hydrocarbon.

Temp. t.	Sp. Grav. d.	Index A. μ_A.	Index H. μ_H.	Angle A. θ_A.	Limit. ν.	Mol. dist. h.	Specific Limit. $\frac{\nu-1}{d}$.
6°·5	0·8926	1·4728	1·4988	11° 4′ 50″	1·4632	......	51893
25·0	0·8820	1·4650	1·4901	10 58 0	1·4561	......	51712
Benzene.							
2·0	0·8979	1·5021	1·5460	13 56 40	1·4873	·039219	54272
23·7	0·8760	1·4893	1·5320	......	1·4725	·039122	54328
28·6	0·8709	1·4860	1·5279	13 50 35	1·4720	·038892	54196

Table III. (*continued*).

Temp. t.	Sp. Grav. d.	Index A. μ_A.	Index H. μ_H.	Angle A. θ_A.	Limit. ν.	Mol. dist. h.	Specific Limit. $\frac{\nu-1}{d}$.
			Benzene.—Another Specimen.				
10°·0	0·8868	1·4935	1·5355	13° 43′ 10″	1·4792	·038802	54037
21·5	0·8773	1·4887	1·5304	13 41 45	1·4745	·038850	54077
			Styrolene.				
11·0	0·9409	1·5208	1·5693	14 1 0	1·5057	·038848	53747
			Another Specimen.				
							53853
			Cresol.				
23·0	1·039	1·5316	1·5787	14 45 0	1·5137	·040776	49442
			Metacresol.				
19·0	1·033	1·5257	1·5726	14 13 50	1·5102	·039395	49390
			Benzil Alcohol.				
22·0	1·0412	1·5278	1·5710	13 58 0	1·5150	·038089	49467
			Methyl Citraconate.				
15·5	1·1164	1·4442	1·4721	11 32 0	1·4345	......	38920
			Methyl Mesaconate.				
16·0	1·1246	1·4492	1·4813	12 19 0	1·4380	... ..	38938
			Picoline.				
23·5	0·94093	1·4912	1·5317	13 29 0	1·4774	·038247	50726
			Aniline.				
13·0	1·016	1·5695	1·6336	16 9 0	1·5488	·043342	54016
			Another specimen.				
7·5	1·0322	1·5780	1·6449	16 25 0	1·5565	·043940	53920
			Benzil Chloride.				
7·0	1·099	1·5314	1·5764	13 59 0	1·5160	0·37148	46592
			Chlorotoluine.				
19·0	1·0761	1·5173	1·5613	13 54 0	1·5027	·038701	46717

Table III. (*continued*).

Temp. t.	Sp. Grav. d.	Index A. μ_A.	Index H. μ_H.	Angle A. θ_A.	Limit. ν.	Mol. dist. h.	Specific Limit. $\frac{\nu-1}{d}$.
Acetone.							
*25·5°	0·8117 at 15°	1·3540	1·3706	9° 25′ 0″	1·3479	·024733	42860
Butyric Ether.							
*23·0	0·8778 at 20°	1·3850	1·4669	9 24 0	1·3778	·027995	43039
Allyl Alcohol.							
23·0	0·8563	1·4054	1·4289	10 44 55	1·3695	·025438	46304
Allyl Sulphide.							
11·0	0·8544	1·4531	1·4811	11 30 40	1·4434	......	51837
Ethyl Thiocarbamide.							
18·0	1·0030	1·5040	1·5477	13 13 0	1·4890	......	48794
Orthobromotoluine.							
......	1·4192	1·5502	1·5981	14 3 50	1·5299	......	37340
Compare this with benzil chloride above					1·5160	......	46592
Isobutyl Chloride.							
19·0	0·8626	1·3939	1·4111	8 57 30	1·3882	......	45004
Isobutyl Iodide.							
......	1·5982	1·4874	1·5240	12 53 0	1·4749	......	29717
Mercuric Methyl.							
30·0	2·966	1·5229	1·5683	14 5 0	1·5076	......	17365
Bromopicrin.							
13·0	2·816	1·5736	μ_D 1·5831	14 0 0	1·5246	......	18612

* It will be seen that in both these d is too great.

TABLE IV.

Substance.	Lower Index.	Upper Index.	Angle θ_A.	Limit.	Limiting Wave-length.
Selenium	μ_A. 2·653	μ_D. 2·980	θ_A. 44° 9′ 0″	ν. 2·1353	5295·7
Solid Sulphur	μ_A. 1·9024	μ_E. 1·9527	20° 30′ 0″	1·8621	2663·8
Solid Phosphorus	μ_A. 2·1059	μ_E. 2·1442	20° 40′ 0″	2·0611	2683·8

TABLE V.

Sp. g.	μ_A.	μ_H.	ν.	$\frac{\nu-1}{d}$.
Water. 1·000	1·32924	1·34393	1·3249	3249
Alcohol. 0·7919	1·3585	1·3735	1·3531	44589

XXV. *The Temperature at which Nickel begins to lose suddenly its Magnetic Properties.* By HERBERT TOMLINSON, *B.A.**

IT has long been known that nickel, like iron, begins at a certain temperature to rapidly lose its magnetic properties, and that the critical temperature for the former metal is much lower than for the latter. According to Faraday, nickel loses its magnetic permeability about 330° to 340° C.; according to Becquerel, about 400° C.; according to Pouillet, about 350° C.; and according to Chrystal, about 400° C. Berson, however, seems to have been the first † to publish a curve showing the relation between magnetic induction and temperature right up to the point at which the former ceases practically to exist. The author has also, independently of Berson, drawn up curves of a similar kind which have not as yet been published, and which he ventures to offer to the Physical Society, because they not only supplement Berson's results, but they seem also to partly explain why different observers have obtained such widely differing temperatures for the point of *nil* permeability ‡.

In the axis of a magnetizing solenoid, and perpendicular to the magnetic meridian, was placed a nickel wire, 30 centim. in length and 0·0053 square centim. in section. The solenoid consisted of cotton-covered copper wire of $\frac{1}{20}$ inch in diameter,

* Read February 25, 1888.

† *Ann. de Phys. et de Chim.* vol. vii. (1886).

‡ That is *practically* nil. The experiments of Faraday seem to show that the permeability never *entirely* vanishes.

wrapped in a single layer of 8·25 turns to the centimetre round a brass tube slit throughout its entire length, and having an internal diameter of 2·5 centim. Before being wrapped round the brass tube the wire was well coated with pipeclay moistened in water. Round 10 centim. of the central portion of the solenoid were wrapped 240 turns of another piece of the same wire, also coated with pipeclay, to serve as a secondary coil. The whole was then introduced into an air-chamber and the wet pipeclay allowed to dry, at first slowly at the temperature of the air, and afterwards more quickly at temperatures which were gradually raised to 400° C. The air-chamber consisted of two concentric copper cylinders 40 centim. in length, enclosing between them an annular space about ·5 centim. thick which was filled with fine sand, and was heated by a row of burners placed underneath. The temperature of the air-chamber was calculated from the alteration of electrical resistance of a coil of platinum wire, whose resistance had been very carefully determined at different temperatures up to 100° C., and expressed in terms of the temperature by a formula of the form

$$R_t = R_0(1 + at - bt^2),$$

where R_t and R_0 are the resistances at $t°$ C. and 0° C. respectively, and a and b are constants. The platinum coil was wound double and was placed inside the slit brass tube close to the nickel wire, but insulated from it and the tube by asbestos; the length of the coil was 10 centim., and it occupied the central portion of the air-chamber. To the ends of the coil were hard-soldered stout copper terminal rods, which passed through a wooden cap closing one end of the air-chamber, and served to connect with a Wheatstone-bridge arrangement employed for determining the resistance of the platinum. The other end of the air-chamber was also closed with a wooden cap, through holes in which passed the ends of the wires of the magnetizing solenoid and the secondary coil. After the pipeclay coating had become thoroughly dry the resistances of the primary and secondary coils were tested, and found to be sensibly the same as before winding. The object of coating the cotton covering of the wires with pipeclay was to maintain the insulation, for at the temperatures reached in

some of the experiments the cotton became charred. Whilst the coils lay undisturbed the pipeclay coating served the purpose for which it was intended very well; but when they were removed from the air-chamber on the conclusion of the experiments, both the charred cotton and the pipeclay easily came away from the wire, and so rendered the coils useless.

A pair of primary and secondary coils exactly similar to the first, except that they had no pipeclay coating and no nickel core inside, was kept buried in a box filled with well-dried sawdust and placed about two feet away from the air-chamber. The two primary coils were connected up in series with each other, a battery, a key, a rheostat, and a tangent-galvanometer whose constant had been previously carefully determined. The secondary coils were also connected up in series with each other, with a rheostat, a Thomson's reflecting-galvanometer, and an earth-coil. The earth-coil consisted of seven turns of silk-covered copper wire laid side by side round the circumference of a wooden disk 30 centim. in diameter; the disk was supported with its plane horizontal, but could be turned round a horizontal axis in either direction through an angle of 180 degrees. The ends of the wires round the disk were soldered to amalgamated copper disks revolving in mercury-cups, which last served to connect the earth-coil with the rest of the apparatus in series with it. The two pairs of primary and secondary coils were so arranged that the currents induced on opening or closing the battery-circuit by means of the key exactly balanced each other before the introduction of the nickel into one of them. When therefore the nickel was introduced, the observed deflection produced by opening or closing the primary circuit was entirely due to the magnetic permeability of the nickel; and by comparing this deflection with that produced by turning the earth-coil suddenly through 180° the permeability could be determined in absolute measure, since the vertical component of the earth's magnetic force at the place was known. The earth-coil also served another purpose, namely to secure with the aid of the rheostat the constancy of the sensitiveness of the Thomson-galvanometer. As the temperature of the air-chamber was raised, the total resistance in both primary and secondary circuits increased; but by altering the rheostats in

these circuits until the deflection of the tangent-galvanometer in the one case, and of the Thomson-galvanometer, when the earth-coil was suddenly turned, in the other, became the same as before the heating, constancy both in the magnetizing force and in the sensitiveness of the Thomson-galvanometer was secured.

The mode of proceeding was as follows :—By means of the set of burners underneath the air-chamber the temperature was raised to nearly the highest point which it was desirable to attain; the burners were then adjusted until the temperature was either constant or very slowly rising or falling; this could always be secured by waiting for a sufficient length of time. The rheostat in the secondary circuit was then adjusted until the deflection of the Thomson-galvanometer, produced by suddenly turning the earth-coil, was exactly 80 divisions of the scale. The rheostat in the primary circuit was next adjusted until the required current had been reached, care being taken to begin with a small current, which was increased very gradually to the required amount. As it was the aim of the author to test only the *temporary* permeability * of the nickel, the current in the primary circuit was opened and closed a great many times, until the current induced on closing the circuit became the same in magnitude as that induced on opening the circuit; and then the mean of the deflections produced by ten times closing the circuit was taken to measure the temporary induction. The air-chamber was now allowed to cool a little, and then, as before, after the temperature had been steadied by adjusting the burners, a fresh set of observations was made, and so on until the temperature of the room was reached. As soon as the observations with the lowest magnetizing force had been completed, fresh ones were made with a higher magnetizing force until a magnetizing force of 18·183 C.G.S. units had been reached.

The temporary permeability of the nickel was calculated from the following formulæ :—

* It is impossible to obtain *true* relations between change of temperature and change of *total* permeability by the "ballistic method," because the mere act of changing the temperature shakes out some of the sub-permanent magnetism which the metal may have acquired at previous temperatures.

$$M_f = \frac{4\pi n_1 C}{l}, \quad \ldots \ldots \quad (1)$$

$$M_p = \frac{2NAVd}{DSn_2M_f}. \quad \ldots \ldots \quad (2)$$

In formula (1) M_f is the magnetizing force, n_1 is the number of turns in the primary coil, l the length of the coil, and C the current circulating round the coil.

In formula (2) M_p is the magnetic permeability (ratio of magnetic induction to magnetizing force), N the number of turns in the earth-coil, A the area of the earth-coil, D the deflection produced by suddenly turning the earth-coil through 180°, V is the vertical component of the earth's magnetic force, d the induction-current due to the nickel, S the section of the nickel, and n_2 the number of turns in the secondary coil. C.G.S. units were used throughout. Since the length of the nickel under the magnetizing force was about 350 times the diameter, the effect of the ends is quite negligible. The results are given in the following table, which is supplemented by the curves in fig. 1. In fig. 2 are given Berson's curves for nickel, and in fig. 3 similar curves obtained by Ledeboer* for iron.

TABLE I.

Magnetizing force.	4·959	6·612		9·918		11·571		18·183	
Temperature, in degrees Centigrade, t.	Permeability, M_p.	t.	M_p.	t.	M_p.	t.	M_p.	t.	M_p.
16·5	93·4	19·5	88·8	20	88·6	19	87·6	18	82·0
84·5	127·9	55·0	97·1	54	93 4	47	91·1	60	84·4
117·6	125·7	93	101·0	74	93·4	84	89·8	79	87·6
128·0	125·0	127	97·6	83	94·1	133	95·2	102	90·8
160·0	126·4	160	107·0	102	95·5	178	97·4	143	89·6
200·0	127·9	186	108·1	143	102·9	191	101·5	174	91·6
227·0	130·9	227	120·7	157	103·7	275	105·6	186	92·6
261 0	141·2	279	126·3	204	106·3	286	104·0	254	92·4
282·0	144·1	301	111·4	247	111·8	295	100·9	293	90·2
297·0	142·6	349	60·7	262	112·1	330	63·0	316	72·2
304·0	139·7	......	......	275	112·9	346	54·2		
306·5	125·0	......	......	308	96·3				
317·0	102·9	......	......	340	65·1				

* Both these and Berson's curves are taken from the 'Electrician,' vol. xx. No. 505.

Fig. 1.

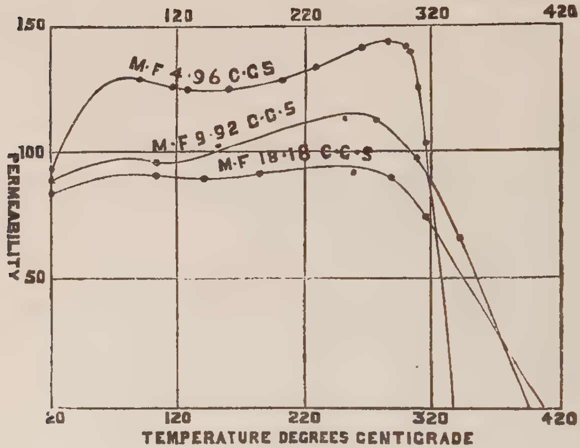
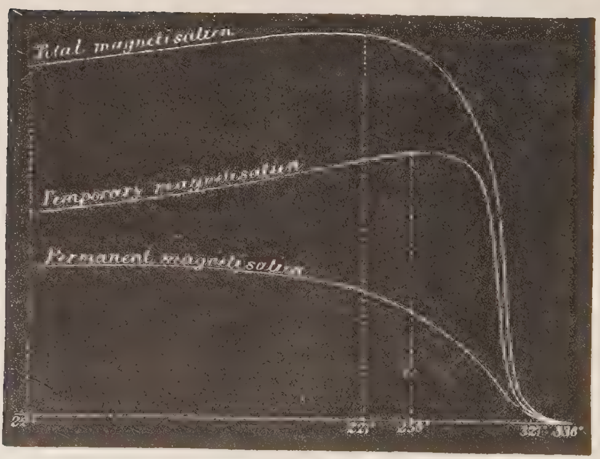


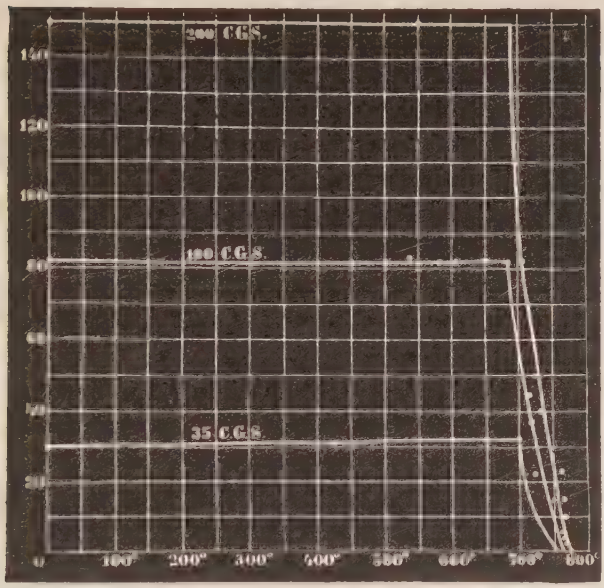
Fig. 2.



It seems from the above table and the curves in fig. 1 that the temperature of maximum permeability is lower the greater the magnetizing force. Thus, for the magnetizing forces 4·959, 9·918, and 18·183, we get the temperatures of maximum permeability as 287°, 248°, and 242° C. respectively.

The temperature at which the permeability practically vanishes seems, on the other hand, to be higher the greater

Fig. 3.



the magnetizing force. As for the above-mentioned magnetizing forces, we have for this temperature the values 333°, 392°, and 412° C. respectively. In this respect the behaviour of nickel resembles that of iron, as is evidenced by the curves in fig. 3; and it is probably partly for this reason that different observers have obtained different results for the temperature of *nil*-permeability. Another cause for the discrepancies in this respect between the observations of different experimenters is probably to be found in the want of purity of the metal; for it is evident that if iron be present as an impurity, the point of *nil*-permeability may be rendered very much higher*. The observations made by the author were not in any case carried right up to the vanishing-point of the permeability, as the nickel used by him was, though nearly, not quite pure†;

* The temperature at which the permeability of iron vanishes seems, from Ledeboer's curves, to lie between 750° and 770° C.

† The wire was procured from Messrs. Johnson and Matthey, who informed the author that they found it impossible to draw pure nickel wire. Mr. G. S. Johnson, the Demonstrator of Chemistry at King's College, London, has kindly furnished an analysis of the wire. It contains 97·5 per cent. of nickel and only 0·67 per cent. of iron.

and it is not at all unlikely that, had this been done, the terminations of the curves would more nearly resemble those of Berson, shown in fig. 2, where it will be noticed that there is a point of inflexion before 320° C. The author does not believe that, even with perfectly pure nickel, the magnetic permeability would *entirely* vanish; but the following experiment will show that, with a thin layer of pure nickel, the permeability *practically* vanishes with great suddenness:—A brass wire 1 millim. in diameter was coated with a thin layer of nickel by electrolysis, and a piece of it, about 2 centim. in length, was suspended horizontally in a cradle of platinum between the pole-pieces of a powerful electromagnet in a direction perpendicular to the lines of magnetic force; the platinum cradle was supported by a platinum wire 12 inches long and $\frac{1}{100}$ inch in diameter. The nickel-plated brass wire was heated by a burner to a temperature of visible red, and the electromagnet was then excited by five Grove-cells. Not the slightest effect of the intense field seemed to be experienced by the nickel, even after it had been cooling for several seconds. The magnetizing circuit was now broken, leaving only the residual magnetism in the pole-pieces, when, after a few seconds of further cooling, and without *the least preliminary warning*, the wire set axially with startling rapidity.

The chief aim of the author in this research was to fix approximately the temperature at which the rate of loss of permeability begins to be greatest; and this appears to be 300° C., not only for the specimen of nickel used by him, but also for that used by Berson, and this, too, both for the temporary and total magnetization. The following table shows the rate at which the temporary permeability decreases with rise of temperature between 300° and 320° C.

From this table it is apparent that the rate of decrease of permeability is less the higher the magnetizing force; so much so indeed that whereas, with a magnetizing force of about 5 C.G.S. units, there is for each degree rise of temperature a decrease of 11 lines of force per square centimetre, with a force of 18 the loss of lines per degree is only increased to 14.

TABLE II.

Temperature, in degrees C., t.	Magnetizing force, M_f.	Permeability, M_p.	Differences between values of M_p at 300° and 320° C. divided by 20, $\frac{\Delta M_p}{\Delta t}$.	$\frac{\Delta M_p}{\Delta t} \times M_f$.
300 320	4·959	141·0 96·6	2·22	11·1
300 320	11·571	95·5 73·8	1·08	12·5
300 320	18·183	84·8 69·1	0·79	14·3

XXVI. *Experiments with Soap-bubbles.* By C. V. BOYS, *A.R.S.M., Demonstrator of Physics at the Science Schools, South Kensington*.*

[Plate VII.]

THOUGH none of the experiments I am about to describe depend upon any property of a soap-film which is not perfectly well known and understood, yet they serve to illustrate in a striking and beautiful manner the behaviour of bubbles under special circumstances, and so as lecture-experiments simply I hope they may be considered worthy of the attention of the Physical Society.

Everyone is familiar with the fact that a soap-bubble may be supported or even struck by a piece of baize or wool without coming into real contact with the material; it is also well known that two bubbles supported on the pipes from which they are blown, or on rings, may be pressed or knocked together with such violence as to materially alter their shape, and yet they do not come into real contact; there is a film of air between them which they are unable to squeeze out. This film, though thin to ordinary tests, is so thick that the colours

* Read April 14, 1888.

of Newton's rings are only seen when one of the bubbles is very small, so that the air is squeezed out the more readily. If the pressure is increased so as to make a real contact, the bubbles both instantly burst. That this pressure may be made great before the true contact takes place will be shown in a variety of ways hereafter; but the following simple experiment makes it very evident that the air-film will prevent the contact of two soap-films that are pressed together.

Exp. 1.—Blow a bubble about 9 cm. in diameter, and place it on a ring with a diameter of about 7 cm. This bubble may be pulled or pushed through the ring by means of a smaller wire ring which serves as a handle. (See 'Nature,' 1871, p. 395.) It may be so adjusted that the weight of the ring will not pull it through. Then a ring larger than the bubble, carrying a plane film, can be used to push it up and down through the ring, and yet the two films do not touch (fig. 1, Pl. VII.).

Bearing this fact in mind, that two bubbles may press one another without true contact, I hoped to be able to blow and detach one bubble within another, and let it roll about within the larger bubble. This, however, is made difficult by the accumulation of a small quantity of solution at the bottom of each, the weight of which is able immediately to press through the air-film between them and so cause both bubbles to burst. However, the experiment can be performed in the following manner:—

Exp. 2.—Blow a bubble on the lower side of the same ring that was used in Exp. 1, and if a large drop does not remain hanging to the bubble slowly apply solution to any part until as great a drop as can safely be carried has accumulated. Then pass the end of the pipe through the upper side of the bubble, and blow another inside, but take care in this case to have no excess of liquid. When the inner bubble is about twice as large as the outer one was at first, remove the pipe with a rapid movement. The inner one will now fall gently and rest within the outer one, the heavy drop pulling the thick part of the outer bubble out of reach of the inner one. The air of the outer bubble may then be withdrawn until the space between the highest point of the

two bubbles is no more than two or three millimetres (fig 2, Pl. VII.).

The great pressure which the air-film will carry is well shown by the next experiment, which, moreover, is more easily carried out than the last.

Exp. 3.—Proceed as in the last experiment, but instead of making a large drop on the first bubble, hang on a moistened ring of wire rather smaller than the fixed ring. This ring should be weighted until it pulls the bubble so much out of shape that a tangent to the curve at the points where the film meets the hanging ring makes an angle of 20° or 30° with the plane of the ring (fig. 3, Pl. VII.). A bubble may then be blown inside and allowed to drop, when it will be found to rest on the conical seat provided by the outer bubble, while the heavy drops of liquid are kept apart, and thus there is no fear of contact (fig. 4). These drops may now be both removed with the end of the blowpipe; then, if the lower ring is pulled down slowly, it will be found that the inner bubble is being squeezed out of shape until it becomes a beautiful oval, while the outer bubble shows the effect of the pressure by a corresponding enlargement (fig. 5). If the lower ring is pulled down still further, the outer bubble is simply pulled in half, and the inner one, often unbroken, gently floats away. This shows that contact was not made, as in that case both would be immediately broken. If, however, instead of pulling the ring too far, it is held in the position shown in fig. 5, it will be found that it is possible to swing the pair of bubbles round and round, and yet in spite of this violent treatment the bubbles refuse to touch one another. Or, if the lower ring is cautiously inclined and pulled away, the outer bubble will peal off it and remain attached to the upper one only. The two bubbles will now be spherical again, but there will be no heavy drop as in fig. 2. The air of the outer bubble may be withdrawn as before, until the two bubbles are barely separate.

This experiment, and many of those that follow, may be made more beautiful by using for the inner bubble a solution strongly coloured by fluorescine, or still better by uranine (for the knowledge of which I am indebted to Mr. Madan); then, if sunlight, electric or magnesium light is thrown on to

the bubbles, the inner one appears a brilliant green, while the outer one remains clear as before.

The power of the surface-tension to do work is demonstrated by blowing a large bubble below the ring and hanging on the weighted ring. If now a very small ring, a centimetre or more in diameter, is placed on that part of the bubble which is stretched across either ring, and then the part within the small ring is made to burst, the air will escape through the small hole and the heavy ring will be lifted until it comes in contact with the upper one. If the film over the whole of the heavy ring is burst instead, the ring is pulled up so suddenly that it is difficult to follow it with the eye, and it strikes the upper ring with such violence that the noise is loud enough to be heard across a large room.

A suspended ring affords a simple and accurate means of measuring the surface-tension of the soap-film. A plane film is formed across a fixed horizontal ring and a light smaller ring is attached to the plane film, which is then broken within the smaller ring so as to leave an annular film only. Weights are then hung on to the suspended ring until the angle between the film and the plane of this ring approaches 90°. At this point equilibrium becomes unstable, and the lower ring falls away, but now both rings will be found to carry plane films, though the moment before neither did. On repeating the experiment a few times it will often be found possible to use such a weight that the ring will hang for some time, but will gradually sink, while the angle referred to above will approach more and more nearly to 90° as the surface-tension of the film diminishes; and thus the exact surface-tension at the particular moment of separation may be found by dividing weight of the ring and attached moisture by twice its circumference.

Exp. 4.—Bubbles blown with coal-gas are lighter than air and rise. If therefore an inner bubble is blown with such a mixture of air and gas as to rise, it will rest against the upper side of the outer bubble, where there are no heavy drops but where the films are thinnest and cleanest (fig. 6). A pair of bubbles blown in this way will sometimes last an hour when exposed to the air of the room. The inner bubble may

be gradually enlarged by blowing in gas until the outer one can barely withstand the pull. The forms assumed under these circumstances are extremely graceful, and their beauty is increased by the play of colours on the two bubbles which the multiple reflexions seem to intensify. If, when the inner bubble is not too large, as in fig. 6, a little gas is gently let into the outer bubble, it is possible to so adjust the mixture of gas and air that the inner will float either near the top or near the bottom of the outer bubble, or about the middle, as may be desired (fig. 8). If under these conditions the bubbles are left undisturbed, the richer gas above the inner bubble will diffuse into the poorer and heavier gas below, and the bubble will slowly rise or fall, according to the relative quantities of gas and air. The diffusion through the film is well shown in the next experiment.

Exp. 5.—Blow a pair of bubbles, as shown in fig. 6, but make the inner bubble only just light enough to rest against the top of the outer one. Lower a bell-jar over all, and pass a stream of gas into the bell-jar by means of a tube passing through the top. As the air is gradually driven down, the outer bubble begins to feel the want of buoyancy, and gradually settles down, as shown in fig. 9. After a short time, the effect of the diffusion through both bubbles tending to enrich the gas of the outer bubble is made evident by the gentle descent of the inner bubble. If the jar is raised quickly, and a little air is blown into the outer bubble, it is possible to again cause the inner one to rise and float against the top of the outer one as before. The bell-jar may be lowered and the process repeated until the outer bubble is so large that the ring is unable to support its weight when in an atmosphere of coal-gas.

The very rapid diffusion of a vapour which will mix with the solution of which the film is made is easily shown.

Exp. 6.—Into a large inverted bell-jar pour a small quantity of ether, or to fill the jar with the vapour quickly wet a piece of blotting-paper with ether and stand it on edge in the jar. Remove the paper, then blow a bubble and drop it into the jar. The bubble will rest on the ether vapour as on carbonic anhydride, and while floating the most violent

agitation of the colours of the film will be seen. The bubble does not remain floating long at the same level; it gradually sinks into denser and denser layers of vapour until it reaches the bottom or breaks on the way. This gradual sinking is due to the penetration into the bubble of the ether vapour, as may be shown as follows:—The bubble may be taken out of the vapour by means of a ring wetted with soap-solution and carried to a flame, when instantly there is a blaze of ether vapour a foot or more in diameter. That the flame is not due to liquid ether condensed on the film is shown by exposing a plane film to the vapour and carrying it to a light in the same way, when no trace of flame will be seen.

Exp. 7.—At the end of a wide tube, which has been enlarged at the lower end, blow a large bubble and lower it gently into the vapour of ether, holding the finger at the mouth of the tube. After a few seconds it will be found difficult to remove the bubble by means of the tube, because its weight may have become sufficient to tear it away when buoyed up by the air only. If it is removed successfully it will hang like a heavy drop; then, on removing the finger, a light may be put to the issuing vapour, which will burn like a bunsen-burner. If, moreover, the bubble full of ether vapour is held in a brilliant light, the shadow will show the ether vapour oozing through the film and falling away in a heavy stream (fig. 10). This experiment shows in succession the floating of an air-bubble on a heavy vapour, rapid diffusion of a soluble vapour through a soap-film, and the power of the surface-tension to force the heavy vapour up a tube fast enough to supply a large flame.

Exp. 8.—Blow a bubble with oxygen gas in a jar partly filled with ether vapour; on taking the bubble out of the vapour and carrying it to a light, it will explode with a loud report. Sufficient vapour will penetrate the bubble, even whilst it is being blown, to make the mixture violently explosive.

Exp. 9.—The weight of the air is well shown by blowing a bubble with gas on a ring and then trying to blow an air-bubble within it (fig. 11). The inner bubble is then pulled out into a pear-shape, and very soon breaks away from the pipe on account of its great weight.

Exp. 10.—If Exp. 4 be repeated, but instead of a

heavy fixed ring a light aluminium one be used instead, to which is tied a long piece of thread which may have a sheet of paper at the end, then the whole combination will float and rise in the air, even though, as in fig. 7, practically the whole of the buoyancy is due to the gas in the inner bubble. In this case the inner bubble is the bag of a balloon, the outer bubble is the netting, and the wire and the things carried by it are the car. In this case the power of the air-film to resist contact of the two films is more evident than ever. If any of the former figures 6, 7, or 8 are carrying a wire ring and thread, as described, it is possible by a suitable pull at the thread to release the pair of bubbles, which float away, one inside the other, until the ceiling brings the experiment to a conclusion.

Exp. 11.—If the inner bubble of fig. 6 is made smaller than the ring, then the corresponding experiment to that represented in fig. 5 is shown in fig. 12. The small sphere will always roll to the upper end of the outer bubble, which may be pulled out to the cylindrical form and be inclined either way. This modification of the other experiment was suggested by Mr. Newth, to whom I had shown the previous combinations.

A great many experiments may be shown in which strings of two or more bubbles, filled some with air and some with gas, tend to pull in different directions. Thus an air-bubble with a gas-bubble blown on the top of it will rise till the gas-bubble breaks against the ceiling, when the air-bubble falls again, and may be sent up as often as desired by the addition of a new gas-bubble; or three bubbles, the lower one of air the upper one of gas, so proportioned that the combination just floats, will remain until the middle bubble is touched with the finger, when the other two immediately go opposite ways. There is no occasion to say more about experiments of this type.

Exp. 12.—An experiment which is easily performed shows in a striking way how the air-film resists being broken. If a pair of bubbles are blown as shown in fig. 4, and the vibrating prongs of a large tuning-fork are brought quite close to the line where one bubble rests upon the other, both films will take up the movement of the fork, and a point of light

reflected by the two films is seen spread out into a pair of rings, so violent is the motion, yet the films do not touch. It is hardly possible to suppose that the two films remain as close together where the movement occurs as in other parts of the line of support; if they tend to separate they form an exception to the general rule that a vibrating body attracts an object in the immediate neighbourhood. In this case the inner bubble is heavier than the air in the outer one, both because of the weight of the film and the compression of the air within due to its tension. But if the same experiment is tried when the inner bubble is lighter than the air in the outer one, as it may be by holding one of the prongs close to the highest point of the bubbles shown in fig. 6, or when either bubble is heavier or lighter than air, the same result will be found—the bubbles will refuse to touch one another.

Plateau has described (*Statique des liquides*) a number of very beautiful experiments in which wire frames representing the edges of geometrical solids are dipped in soap-solution, after which they are found to carry combinations of films, plane or curved according to the character of the frame. Thus within a triangular prism, when it is removed from the solution, is found a combination of nine plane films which form three troughs meeting along the axis of the prism and a triangular pit at each end.

Exp. 13.—A spherical bubble may be dropped into one of these troughs and rolled from end to end, it may be taken out of one trough and dropped into another, or the frame may be held with its axis vertical, when the bubble may be dropped into the triangular pit, where, however, it will not remain long.

The characteristic feature of all the laminar figures is that there are never more than three surfaces meeting in a line where the angles are always 120°, or more than four lines or six surfaces meeting in a point. Further, the mean curvatures of the films are always zero so long as no air is enclosed. As Plateau mentions in his book, the screw-surface has no mean curvature, therefore if a frame be made out of a helix of wire with its ends connected by wire to a solid axis, such a frame after being dipped will carry a helical surface of soap-film.

Exp. 14.—If, instead of a single helix of wire, two helices are fixed to the same axis, but not quite symmetrically, so that in any part the wire of one helix is nearer that above it than the one below, two helical films will not be formed, but there will be a single one in an intermediate position which will be joined to the two wire screws by a pair of conical screw-surfaces, these forming with the true screw-surface a screw-shaped trough down which bubbles may be rolled or up which they may be wound, as water is wound up by a screw-pump (fig. 13). Further, if a series of small bubbles are blown along the helical edge in which the three films meet at angles of 120°, a spiral staircase is made of soap-film, down which a bubble will run one or two steps at a time, and from which it will escape uninjured when it reaches the bottom. Of course bubbles lighter than air, in the same way, will rest against the lower sides of a trough or roll up instead of down the screw-surface.

Exp. 15.—One more experiment in which the rolling of bubbles is the chief feature is worth describing. Three rings of wire, seen in section in fig. 14, are joined together by wires, shown dotted, and are carried by a central axis, which may be made to rotate. After this frame has been dipped in the solution of soap, and the three radial planes broken, it is found to carry a circular trough, into which a series of bubbles may be dropped, while at the same time the frame may be kept rotating, so that the bubbles are rolling round and round like marbles on the rim of a solitaire-board. A corresponding frame might possibly be made of light wire, which after dipping would rest on the bubbles in the first frame, thus forming a working model of the ball-bearing. I have not, however, succeeded.

Plateau has mentioned the fact (p. 166) that M. Chautard has found a soap-film a convenient envelope for a gas which is to be tested magnetically. He says that a spherical film above one pole of an electromagnet is visibly disturbed if the gas within has magnetic properties when the exciting current from a battery of 25 to 30 Bunsen's cells is made and broken. If, instead of a spherical bubble, one of cylindrical form, with its length about three times its diameter, is used, the distortion produced by a small disturbing force is so greatly

magnified that, using an electromagnet actuated by five Grove's cells only, not only is the change of form manifest when oxygen is the gas in the bubble, but it is even possible, by making the length such that the form is very nearly unstable, to cause the bubble to divide the moment the current is made to pass round the electromagnet. With the same means I have not been able to detect any change of form in a spherical bubble. Fig. 15 shows a convenient apparatus for producing, as often as may be desired, a cylindrical bubble of any degree of stability.

The short tube *a* is in connexion with a supply of oxygen which is employed to blow the spherical bubble shown by the dotted circle. According to the position of the screw *d* this bubble will be larger or smaller before it comes in contact with the ring *b*, which is held down by the loose weight *w*. The gas-tap is then immediately turned off, and the ring *b* raised by the action of the weight *c*, until the screw *e* brings the movement to a stop, Thus the length of the cylinder depends on the screw *e*, while its volume is determined by the screw *d*, and so whatever degree of stability is found suitable can be reproduced as often as may be required. The poles of the magnet should be placed at about the level of the line *pp*.

There is one other property of a pair of soap-films resting against one another, but not in contact, to which I have not referred. In a lecture at the Royal Institution a few years ago Lord Raleigh showed that two water-jets if perfectly clean will, if directed so as to meet one another at a small angle, be reflected again and fall as two separate jets, never really coming into contact at all. If the water is not perfectly clean, the experiment will not succeed. He showed that such a pair of mutually reflected jets form a very delicate electroscope, so that if a piece of excited sealing-wax is held even at a considerable distance they instantly coalesce. As the two jets in his experiments and the two bubbles in those which I am about to describe are in each case kept apart by a thin film of air, I expected to find a pair of bubbles attached to two rings in the same way act as a delicate electroscope.

Exp. 16.—If a pair of bubbles are blown on rings, which

must be insulated from one another, as shown in fig. 16, and the cover of a small electrophorus is raised even at some yards distance, instantly the two bubbles coalesce as seen in fig. 17, but do not burst as they have hitherto been found to do. Or if the two rings are connected with a key and a single bichromate-cell so that when the key is not pressed the rings are connected together, but when depressed they form the terminals of the cell, then at the moment of making the contact the bubbles unite because the electrostatic attraction between surfaces so very close together is able to squeeze out the air, which mere pressure had hitherto failed to do.

Exp. 17.—Bearing in mind how exceedingly delicate this is as a test of difference of potential, the following experiment seems the more decisive. The cover of the electrophorus may be brought so close to the side of the bubbles, shown in fig. 4, as to pull them completely out of shape, and yet the outer film so completely screens the inner from the electrical action, even though the inner one is to all appearance in contact with the outer one, that there is no difference of potential between them, and so the film of air is not destroyed. I do not know any experiment which so clearly shows as this that electrical force is confined to the absolute surface of a conductor, and is not felt at any depth within it however small.

Plateau has mentioned, p. 168, that a hemispherical film blown on a plate will screen a smaller hemisphere blown within it, and also resting on the plate, from electrical disturbance ; but in this case the two films are widely separate, and there is not the same delicate test as in the case of two bubbles apparently coincident, which instantly join when the smallest electrical stress exists between them.

Exp. 18.—One more experiment, which is a combination of these two, is worth performing. If one of the bubbles of fig. 16 is replaced by the combination shown in fig. 4 while the other remains as before, and if the cover of the electrophorus is raised anywhere in the neighbourhood, immediately the two outer films join and become one, while the inner bubble undamaged and the heavy ring slide down to the bottom of the now enlarged single bubble, and give rise to the form shown in fig. 18.

I am perfectly sensible of the fact that these experiments lie very closely on that ill-defined border-line which separates scientific work from scientific play, but I trust that the beautiful way in which they illustrate the action of certain forces may be sufficient excuse for my showing members of the Physical Society what cannot fail to remind some of them of their nursery days.

The following particulars may be of service to those who wish to repeat any of these experiments.

The solution that I have used is composed of

1	part by weight	oleate of soda.
40	"	distilled water.

These, when solution is complete, are well mixed with one third the volume of glycerine and left for a week to settle in stoppered bottles. The liquid is then syphoned off from the impurities which have risen to the surface and clarified with a few drops of ammonia.

The thick wire rings and frames are made of tinned iron wire $1\frac{1}{2}$ millim. in diameter, well cleaned with emery cloth.

The thin wire rings may be made of any thin wire, but aluminium about $\frac{1}{2}$ millim. in diameter does well.

I have found it necessary to make a blowpipe with a trap as shown in fig. 19 to catch condensed moisture, which is apt to cause a failure if it mixes with the bubble. The diameter of the mouth at *a* is 7 millim. For detaching small light bubbles a pipe with a smaller mouth should be used.

When both gas and air are used in any experiment and it is necessary to regulate the proportions very carefully it is well to have a T-piece attached to the blowpipe, so that either gas or air may be blown or stopped at pleasure.

XXVII. *On the Use of the term "Resistance" in the Description of Physical Phenomena.* *By* R. H. M. Bosanquet *.

The following observations were suggested by a remark quoted by Prof. S. P. Thompson from Dr. Hopkinson, at the meeting of the Physical Society on January 28, 1888. The remark was to the effect that there is no such thing as magnetic resistance, because the resistance of iron to magnetization is not constant under varying conditions. I wish to bring before you the question, What is the fundamental idea involved in the use of the term "resistance" in the description of physical phenomena?

If we apply force to produce any change, say to extend an extensible body, or compress a compressible one, the body is said to offer resistance to the force. If we have a second body which requires a greater force than the first to produce the same change, we say that the second body offers a greater resistance than the first. Consequently resistance increases with the force employed to produce a given change.

Again, if in the second body the same force produces a greater change than in the first, the resistance of the second body is said to be less. Consequently resistance diminishes as the change produced increases, and *vice versâ*.

The change may consist in the flow of a fluid, through a tube say, under pressure; then we should say that the resistance offered by the tube to the flow increases as the pressure increases and as the flow diminishes.

Or, again, we may consider a liquid being evaporated under the action of heat. We may say that the resistance to evaporation is the greater the more heat is required and the less liquid evaporated.

We may shortly refer to the two elements in any such change as the cause and the effect. And we shall generalize the conception of resistance in stating that it is the greater the greater the measure of the cause and the less the measure of the effect. But nothing is so far assumed as to the precise law which connects these quantities.

If we now refer to the case of electrical resistance, we

* Read February 11, 1888.

have a law suggested by Ohm's law, viz., that the measure of the resistance is the direct ratio of the measures of cause and effect. Or, to take the words "measure of" as understood, resistance is the ratio of cause to effect, or the quotient of cause by effect; or, resistance is proportional to the cause and inversely as the effect.

This is not the least necessary. We might very well, if there were any reason for it in any particular case, adopt as the measure of resistance such a function as the square, or the square root of the above ratio. It would be quite compatible with the general definition deduced above from the meaning of the word. But Ohm's law ties us down to the direct ratio.

If, therefore, we extend the analogy of Ohm's law to other cases, we shall have some such propositions as follows:—

If force be employed to extend an extensible body or to compress a compressible one, the resistance to extension or compression, on the analogy of Ohm's law, is the quotient of the cause, or force employed, by the effect, or amount of change.

If liquid flow through a pipe under a difference of pressure between its ends, the resistance, on the analogy of Ohm's law, is the quotient of the cause, or difference of pressure, by the effect, or rate of flow.

In such cases the precise manner in which cause and effect are measured must be determined by convenience. It does not in the least affect the conception of resistance as the ratio of cause and effect.

Again, if a liquid be evaporated under the action of heat, the resistance to evaporation, on the analogy of Ohm's law, is the ratio of the cause, or heat expended, to the effect, or amount of liquid evaporated, no matter how these are measured.

We may add a further illustration. Suppose that any substance is heated. Different substances will rise to different temperatures in virtue of their different specific heats. Here the resistance to heating up, on the analogy of Ohm's law, is the ratio of the cause, or heat expended, to the effect, or rise of temperature produced, no matter how these are measured.

In all these cases we have a perfectly intelligible physical conception of resistance, strictly in accordance with the natural meaning of the word in the English language.

But I think we should not apply by way of objection in such cases arguments such as the following :—*i. e.* (1) That the resistance so measured is not of the same dimensions as electrical resistance ; or (2) that the resistance so measured is not a constant, but liable to variation depending on the conditions, as well as to complication by other factors.

The ground that underlies the employment which has been made of these two arguments in opposition to the use of the term resistance in connexion with magnetism, is really the assumption that the same word ought to imply identity in the two cases of electricity and magnetism, and not analogy. If it be admitted that analogy is sufficient ground for the use of a word, the objections disappear.

But if used in connexion with two different subjects, it is quite impossible that the meanings of the word can be identical. And that there is no objection in practice to the use of the same word in meanings justified by analogy only may be shown by many illustrations. Take the case of the word Potential. It originated I suppose in connexion with mechanical theory. It has a definite mechanical meaning, and definite dimensions regarded as a mechanical quantity. But we extend the term without difficulty to both the magnetic and electrical analogies ; and in each of these cases it has another definite meaning, and other definite dimensions. Yet nobody thinks of objecting to either of these uses of the term potential, because they involve definitions of the dimensions of potential, different both from that used in mechanics and from each other.

Similarly with the word Force. This is a purely mechanical term. But nobody objects to its use in the terms electromotive force, or magnetizing force because these uses involve definitions of the dimensions of Force different both from that used in mechanics and from each other.

It is fully understood in these cases that analogy is all that is involved, and nobody supposes that there is any question of identity.

I will now for a moment compare the case of magnetic

resistance with a natural case of ordinary resistance, to show that the analogy between the two is extremely close.

Consider a mass of matter in any condition; suppose it enclosed in a vessel and subjected to compression. Then the cause will be the pressure, the effect the compression, and the resistance on the analogy of Ohm's law the ratio of pressure to compression. If we choose to measure the compression by the inverse of the volume, the resistance will be the product of pressure and volume.

If we suppose the matter to be at first gaseous, and the compression to take place with loss of heat according to Boyle and Marriotte's law, the resistance will be so far constant. But as the compression advances, and the liquid and solid states are approached, the resistance will necessarily increase, and when the compression reaches a certain value it will be practically incapable of proceeding further, and the resistance will increase indefinitely with the pressure. The resistance is therefore here a function of the compression, *i. e.* of the effect, just as magnetic resistance is a function of magnetic induction; and, if we omit the initial part of the curves of magnetic resistance of bars, the rest of them is oddly similar to the course of the resistance in the case imagined.

Now the resistance in the case imagined (of the compression of matter) is not constant, but is a function of the effect just as magnetic resistance is. Is this want of constancy an objection to the use of the term resistance as a description of that quality in matter which tends to prevent compression? For if it is a sufficient objection to the term magnetic resistance, it must be a sufficient objection to the term resistance to compression as well, as the analogy between the two cases is so extraordinarily close. It cannot be maintained for a moment that the want of constancy offers any objection.

An objection may be taken possibly that the magnetic resistance has varying values for rising and falling magnetizing force, and is therefore not definitely ascertainable as a function of magnetizing force. Answer: We do not give up trying to find out the true values of things because they are superficially complicated with others. If magnetic

retentiveness does lead to complications, it is our business to disentangle them; and practically the method of reversals of magnetizing current enables us to do this. But it is not maintained, at least by me, that magnetic resistance is properly a function of magnetizing force. It is a function of the effect produced, *i. e.* of the magnetic induction, as is amply proved by the inspection of the numerous curves that have been published in connexion with this question. And it is so just in the same way in which the resistance of matter to compression is a function only of the compression, or of the stage which the condensation has reached.

A word may be usefully said here on the origin of the differential formula used by Lamont to express the so-called magnetic conductivity of a magnet. The ambiguity on which the origin of this formula depends can be well dealt with by the present illustration.

If we consider the compression of matter which has attained the liquid condition, we may speak of the resistance to any further compression at this point, or under these conditions, as being infinite. For the effect produced by the further pressure is nothing. But in the magnetic analogy we always consider a change, the initial condition of which corresponds roughly to the gaseous condition, or rather a pregaseous condition, in the material analogy. In either case, if, instead of considering the total change, we consider the state of things at a point, then we must suppose the cause to vary by a small quantity, *d* cause, producing a small effect, *d* effect; and the ratio of these may be said to be the resistance at the point of the representative curve, or under the given conditions. It is on this mode of statement that Lamont's differential formula is based. The formula is (*Handbuch des Magnetismus*, p. 41),

$$dm = k\,(\mathrm{M} - m)\,dx;$$

whence by integration,

$$\mathrm{M} - m = \mathrm{C}e^{-kx};$$

x = magnetizing force,
m = magnetism,
M = maximum of magnetism.

Here dm/dx is called the magnetic conductivity; and it is

inversely as the *magnetic resistance at a point.* That magnetic conductivity, which is inversely as the magnetic resistance as I and others use the term, is m/x, i.e. has reference to the *total change.* I have shown (Electrician, xvi. p. 247) that the assumption $m/x = k\,(\mathrm{M} - m)$ is that to which we are thus led, in place of Lamont's formula above given. Prof. S. P. Thompson has shown that this formula, which is the same as that known as Frölich's, corresponds with the facts better than Lamont's.

In what precedes I have, for convenience in quotation, used the ordinary term magnetizing force, and regarded it as a cause. As, however, so-called magnetizing force is identical in dimensions and physical nature with the rest of the magnetic induction which it develops, I use the expression in this way under protest only; for I regard it as inconceivable that cause and effect should be of identical nature, unless the effect reacts again as a cause, so that the smallest original cause drives the effect up to saturation. Without going into detail I may mention an analogy where cause and effect are identical in nature, and the process does therefore necessarily always go to saturation; viz., the case of the multiplication of germs, in a habitat of limited capacity. Here the individuals which formed the original cause multiply; the progeny constitute at once the effect and an increase of the cause. And the smallest original infection of the cause is enough to develop saturation, *i. e.* the highest population that the habitat can maintain. Now in the case of magnetism, magnetizing force and the rest of the magnetic induction are by definition identical in nature. If they are cause and effect, the effect should therefore act again as cause, so as to produce saturation from the smallest force; but this does not occur. I prefer therefore to regard the difference of potential as the cause, as is always done in the case of the electric current.

There is only one further point. It may be maintained that the mention of Ohm's law involves *sub silentio* the assumption that the corresponding law holds true, *i. e.* that the resistance is constant. If this is supposed, it is only necessary to explain that it is not intended. It is clearly legitimate to measure a quantity in the manner suggested by Ohm's law (*i. e.* by the ratio of cause to effect), and speak of the

laws or courses of values thus obtained as representing in the different cases the analogues of Ohm's law. These are the laws which occupy the place in the various matters dealt with, which Ohm's law occupies in the subject of the flow of electricity, *i.e.* they express the ratio of cause to effect in the different cases. But the statement that they are the analogues of Ohm's law does not involve the position that they are identical with it, any more than the definitions of Potential or Force in electricity and magnetism are identical with those in mechanics or with each other.

The main points I have dealt with are :—That resistance may be conceived of quite generally as the ratio of cause to effect.

That the objections to the employment of the term magnetic resistance have been founded on the assumed requirement of identity instead of analogy between the affections of different subjects, and cannot be sustained.

And that the extension of the employment of the term Resistance in the above manner leads to some remarkable analogies which go far to justify independently the employment of the term Magnetic Resistance. And it has been pointed out that the ordinary mode of statement involves inconsistent and impossible ideas as to the relation of cause and effect in the phenomena, whereas the application of the term Magnetic Resistance compels us to precisely ascertain these, and put them in their right places.

Note.—An objection has been recently raised by M. Hospitalier* to my definition of Magnetic Resistance, which comes to this, that in defining the effect I take the induction through unit surface instead of the total induction. This objection is again based on the supposed necessity for conforming identically to Ohm's law in Electricity. The foregoing remarks will have made it clear that I consider this unnecessary, the question being, What is the most convenient measure of the effect? Since the magnetic potential attainable is practically limited, though it increases with the dimension, and the induction through unit of area practically attainable is also limited (by saturation), the ratio of these two quantities lies always within certain limits, though it increases with the dimensions.

* 'Electrician,' xx. p. 164, note.

In fact the magnetic resistance thus estimated is of linear dimension, and may be regarded as of linear scale in plans. If, therefore, this quantity be selected to be tabulated, the discussion of questions of design from a practical point of view is greatly facilitated, and the relations of the quantities involved are more easily followed and more simply expressed.

XXVIII. *The Efficiency of Incandescent Lamps with Direct and Alternating Currents.* *By* W. E. AYRTON *and* JOHN PERRY*

IT is now well understood that in order to economically distribute power by means of electricity it is necessary to employ a high potential-difference, or P.D., between the mains and a small current flowing through them, while considerations of safety require that the P.D. between the leads in the houses shall not exceed 100 or 200 volts. Hence some system of converting a large P.D. and small current into a small P.D. and large current has to be employed; and the four systems of conversion that have hitherto been devised consist in the employment of:—1. Motor-Dynamos, 2. Accumulators, 3. Alternating Current-Transformers, 4. Direct Current-Transformers.

Of these four methods the third is the one that is most extensively utilized at the present time; indeed it is the only system of conversion that is at all extensively used, at any rate in this country. But in view of some direct current system of conversion also coming into common use, there arises a question of considerable importance to the consumer, viz. Is more light obtained with the same expenditure of power with direct or with alternating currents? And apart from considerations of the electric distribution of power on a large scale, the answer to this question is of importance in supplying one factor towards the decision as to the relative advantages and disadvantages of direct and alternating currents for detached installations.

Where the electric energy supplied to a consumer has been charged by meter, people have, as a rule, been content to measure only the number of coulombs supplied, ignoring

* Read February 25, 1888.

altogether any variation in the volts; but as electricity *per se* apart from the P.D. is of no commercial value whatever, and therefore is unlike water apart from pressure, it is clear that in estimating the value of a supply of electric energy for lighting purposes we must measure the watts and not merely the current. The problem, therefore, that we have attempted to solve is whether a Board of Trade unit (1000 watt-hours) is more valuable for lighting by incandescent lamps when the current is direct or when the current is alternating.

For a complete solution of this problem we must ascertain not merely the "efficiency of the lamp" or the candles per watt with the two systems of supply, but also the life of the lamp, since the cost of lamp renewals may be as important a question for the consumer as the bill for electric power. Unfortunately, however, there does not exist, as far as we are aware, any accurate information as to the life of an incandescent lamp with various alternating P.Ds.; indeed the data at our disposal for the life of a lamp with various non-alternating P.Ds. is at most meagre. And if any Member of the Society can supply us with information regarding the life of some fixed type of incandescent lamp either with different direct P.Ds. or with different alternating P.Ds. we shall be grateful for the information, as it will furnish us with a further opportunity of using the method described in our paper read before this Society for deciding on "The most Economical Potential Difference to employ with Incandescent Lamps." *

For the present, therefore, we shall confine ourselves solely to the question of efficiency, and, as it is known that the efficiency of an incandescent lamp increases with the current passing through the lamp, it is clear that to obtain an accurate comparison of the efficiencies with direct and alternating currents, we must employ exactly the same current in both cases, or rather the same mean square of the current, for this will be most likely to develop the same rate of production of heat in the lamp, since this rate is proportional to the resistance into the mean square of the current, whether the current be direct or alternating, and whether or not there be

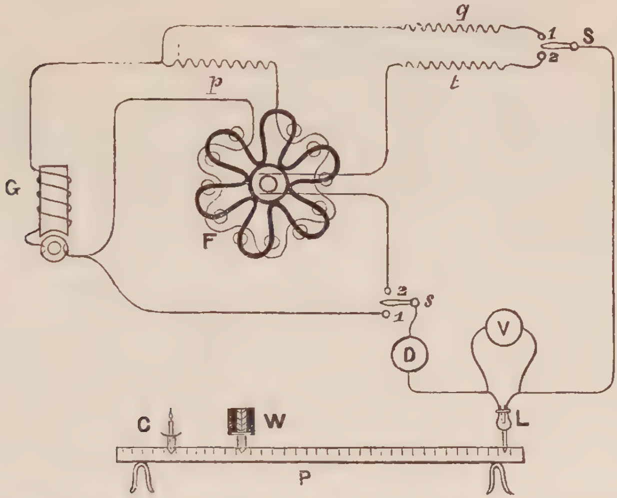
* Proc. Phys. Soc. vol. vii. p. 40.

self-induction in the circuit in which the heat is developed. We say will be most likely to develop the same rate of production of heat, since we must not assume without proof that for the same mean square of current the resistance of a carbon filament is the same whether the current be direct or alternating.

Some writers have stated that the current, as measured by an electrodynamometer, required to be sent through an incandescent lamp when emitting a definite amount of light, had a different value when the current was an alternating one from what it had when the current was direct. This difference might be due either to a defect in the dynamometer measurement or to some variation in the light standard. If the wire with which a dynamometer is wound be thick, then the current-density may be far from uniform when the current is alternating; and on this account, as observed by Captain Cardew some years ago, a dynamometer might give different readings for different rates of alternations of the current, while an incandescent lamp in circuit with it remained equally bright. To avoid this possible cause of error the dynamometer, which was constructed by Messrs. Shepherd, Vignoles, and Wheatley (the three of the students of the Central Institution who carried out the investigation), was wound with much finer wire than would usually be employed in the construction of a dynamometer not required to read currents much below one ampere. The dynamometer was for that reason unnecessarily sensitive, and it required a fairly strong spring to control the motion of the suspended coil. This led to an unnecessary waste of energy in the dynamometer, but that was of no consequence in this investigation, as our object was to measure the mean square of the current most accurately, and not to satisfy the condition, which is of considerable importance in the design of commercial measuring-instruments, of wasting as little energy as possible in the instruments.

To the suspended coil of the dynamometer was attached a mirror, and the values of the deflections of the spot of light were determined by direct comparison with the simultaneous readings of an accurately calibrated magnifying spring-ammeter when various direct currents were successively sent through the circuit. The sensibility was such that a deflec-

tion of 400 scale-divisions was produced on a scale 68·88 inches away, for a current of 2·53 amperes, which corresponds with a deflection of 140 scale-divisions for 1·5 ampere,



and this was about the usual current passing through the dynamometer in the actual lamp experiments The value of the current could, therefore, be accurately measured by this dynamometer, which is indicated by D in the figure, and by knowing what fraction of the current passing through the dynamometer D passed through the non-inductive voltmeter V, the current passing through the lamp is known.

Any error that might have arisen from a variation of the light-standard was eliminated by taking successive readings with a direct current produced by a Gramme-dynamo G, and with an alternating current produced by a Ferranti-dynamo F, the switches S and *s* being turned to 1, 1 in the first case and to 2, 2 in the second. The Gramme-dynamo G was also used to excite the field magnets of the Ferranti, a suitable current being obtained by a proper adjustment of the resistance *p*. By means of the resistances *q* and *t* the direct and alternating currents passing through the incandescent lamp L could be respectively varied; and it was found that if these resistances were so adjusted that the reading of the dynamometer D was the same in both cases, so also was the reading of the non-inductive voltmeter V.

It is known when power is supplied by means of an alternating current to a circuit of resistance r ohms and coefficient of self-induction l secohms that

$$\frac{\text{number of true watts}}{\text{number of measured watts}} = \frac{r\tau}{\sqrt{l^2\pi^2 + r^2\tau^2}},$$

where the measured watts are obtained by multiplying $\sqrt{\overline{A^2}}$, the square root of the mean square of the amperes as measured by the dynamometer, by $\sqrt{\overline{V^2}}$, the square root of the mean square of the volts as measured by the non-inductive voltmeter, and where τ is the time between one alternation and the next, or half a complete period. Therefore the

$$\text{number of true watts} = \frac{r\tau\sqrt{\overline{A^2}\,\overline{V^2}}}{\sqrt{l^2\pi^2 + r^2\tau^2}},$$

where in our case r and l are the resistance and coefficient of self-induction of the carbon filament of the lamp. In the first set of experiments a lamp with a looped filament was employed; but the experiments seemed to indicate that the value of l was not quite small enough to make the term $l^2\pi^2$ absolutely negligible, or else that there was some slight mutual induction between the dynamometer-coils and a small brass vessel containing oil, in which moved a damping-vane attached to the moving coil. The results were therefore discarded, and this vessel was replaced by one made of non-conducting material, and all metal near the dynamometer was as far as possible removed. The lamps subsequently employed were first one with a M-shaped filament, and another with a simple horseshoe-shaped filament, each possessing little self-induction.

The light was measured by comparison with a standard candle, the two being placed 130 centimetres apart on the photometer P, and a screen composed of two pieces of paraffin-wax W, with silvered paper between them, was adjusted until the two pieces of wax appeared equally bright, the comparison being first made when the screen was looked at through ruby-red glass and then through signal-green glass.

The following is a sample of the results obtained, a and b being the distances respectively of the screen from the incandescent lamp and from the standard candle.

TABLE I.

	$\sqrt{A^2}$.	$\sqrt{V^2}$.	Watts.	Green.			
				a.	*b*.	Candles.	Watts per Candle.
Ferranti......	1·34	50	67	107·7	22·3	23·33	2·872
Gramme......	1·34	50	67	105·8	22·2	23·52	2·848
Ferranti......	1·34	50	67	107·9	22·1	23·74	2·823
Gramme......	1·34	50	67	108	22	24·11	2·779
	$\sqrt{A^2}$.	$\sqrt{V^2}$.	Watts.	Red.			
				a.	*b*.	Candles.	Watts per Candle.
Ferranti......	1·34	50	67	106·5	23·5	20·52	3·265
Gramme......	1·34	50	67	106·4	23·6	20·34	3·295
Ferranti......	1·34	50	67	106	24	19·51	3·435
Gramme......	1·34	50	67	105·9	24·1	19·32	3·469

It will be observed that in the set of four successive observations given in Table I. for green light, the candle-power of the lamp appears to be steadily increasing, a result probably due to the brightness of the candle slowly diminishing ; but although this would make the *absolute* determination of the efficiency of the incandescent lamp inexact, it introduces no error in the determination of the *relative* efficiencies of the lamp with direct and alternating currents, since every observation with the direct current was immediately followed by one with alternating current, so that the means of all the corresponding direct-current measurements may be safely compared with the means of all the alternating-current measurements. Similarly for this investigation it is unimportant whether the standard candle used on one day was slightly more or less bright than the standard candle used on the following day. Further, since for any two successive observations with the same coloured light with direct and with alternating current the screen was at practically the same distance from the lamp, the fact that the rays of light coming from the lamp made a somewhat different angle with the surface of the paraffin-wax from the angle made by the rays coming from the candle,

introduced no error in this investigation of *comparative* efficiencies, although it might very likely do so in an absolute determination of the efficiency of an incandescent lamp. In fact the simple device of successively taking observations with direct and with alternating currents throughout the whole investigation removed most of the objections that usually may be made against photometric determinations carried out with a candle as the standard of light.

Disregarding the experiments made with the lamp with the looped filament, for the reason given above, Table II. gives the summary of the results obtained; and the conclusion to be

TABLE II.—Mean Values.

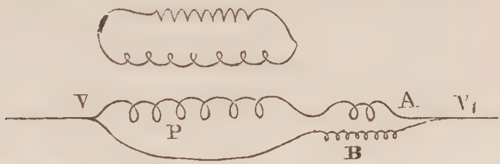
Lamp.	No. of Experiments made.	$\sqrt{\overline{A^2}}$.	$\sqrt{\overline{V^2}}$	Watts per Candle. White Light.				Time of an alternation, in seconds.
50 volt M-shaped filament.	20	1·25	50·5	Gramme. 3·053		Ferranti. 3·033		$\frac{1}{453}$
				Green Light.		Red Light.		
				Gramme.	Ferranti.	Gramme.	Ferranti.	
50 volt horseshoe-shaped filament	19	1·30	50·0	2·597	2·534	3·100	3·100	$\frac{1}{453}$
50 volt horseshoe-shaped filament	20	1·34	50·0	2·935	2·966	3·254	3·164	$\frac{1}{167}$
50 volt horseshoe-shaped filament	16	1·35	50·0	2·900	3·073	3·504	3·477	
Mean of the last three results				2·811	2·857	3·286	3·247	
Mean of all the 75 experiments				Gramme. 3·0490		Ferranti. 3·0497		

In all the experiments the plane of the filament was perpendicular to the screen.

drawn from them is that although the watts per candle for green light are not quite the same for direct and for alternating currents, and although the volts per candle for red light are also not exactly the same for direct and alternating, still the difference between the results for the same colour is so small that it may be put down to experimental errors; and this, combined with the fact that the mean of all the 75 experiments gives practically the same number of watts per candle for both direct and alternating currents, leads to the practical certainty that *the efficiency of an incandescent lamp is the same for both direct and alternating currents.*

XXIX. *On the Measurement of the Power supplied to the Primary Coil of a Transformer.* *By* E. C. RIMINGTON*.

In the discussion on Mr. Kapp's paper on Transformers, at the Society of Telegraph Engineers, Professor Ayrton gave a formula for calculating the true power supplied to the primary from the reading of a Siemens wattmeter. The thick wire coil of the wattmeter is in series with the primary



coil, and the fine wire coil connected as a shunt on the two, as in the diagram, where P is the primary coil, A the thick and B the fine wire coil of the wattmeter. Let r_1 be the resistance of the primary P including A, and L_1 its coefficient of self-induction also including A ; let r_2 and L_2 be the resistance and coefficient of self-induction of B ; i_1 and i_2 the currents in P and B respectively. Let e be the potential-difference between the points V and V_1. Now

$$e = \mathrm{E} \sin at, \text{ where } a = \frac{2\pi}{\mathrm{T}},$$

T being the periodic time, and E = the maximum value of e. Also

$$i_1 = \mathrm{A}_1 \sin (at - \psi_1), \text{ where } \tan \psi_1 = \frac{a\mathrm{L}_1}{r_1},$$

and

$$i_2 = \mathrm{A}_2 \sin (at - \psi_2), \text{ where } \tan \psi_2 = \frac{a\mathrm{L}_2}{r_2}.$$

Let δ be the reading on the torsion-head ; then

$$\delta = \frac{k}{\mathrm{T}} \int_0^{\mathrm{T}} i_1 i_2 \, dt \text{ ; } k \text{ being some constant.}$$

$$\delta = \frac{k}{\mathrm{T}} \mathrm{A}_1 \mathrm{A}_2 \int_0^{\mathrm{T}} \sin (at - \psi_1) \sin (at - \psi_2) \, dt,$$

$$= \frac{k\mathrm{A}_1\mathrm{A}_2}{2\mathrm{T}} \int_0^{\mathrm{T}} \{\cos (\psi_2 - \psi_1) - \cos (2at - \psi_1 - \psi_2)\} \, dt,$$

$$= \frac{k\mathrm{A}_1\mathrm{A}_2}{2} \cos (\psi_1 - \psi_2).$$

* Read March 10, 1888.

Now
$$A_2=\frac{E}{S}, \text{ where } S=\sqrt{r_2^2+a^2L_2^2}.$$

Hence
$$\delta=\frac{k}{S}\cdot\frac{EA_1}{2}\cos(\psi_1-\psi_2).$$

Now mean power, or
$$p_m=\frac{1}{T}\int_0^T ei_1 dt,$$
$$=\frac{1}{T}EA_1\int_0^T \sin at\,.\sin(at-\psi_1)\,dt$$
$$=\frac{EA_1}{2}\cos\psi_1.$$

Therefore
$$p_m=\frac{\delta}{k}\cdot S\cdot\frac{\cos\psi_1}{\cos(\psi_1-\psi_2)}.$$

But
$$\frac{\cos\psi_1}{\cos(\psi_1-\psi_2)}=\frac{\cos\psi_1}{\cos\psi_1\cos\psi_2+\sin\psi_1\sin\psi_2}$$
$$=\frac{\dfrac{1}{\cos\psi_2}}{1+\tan\psi_1\tan\psi_2},$$
$$\frac{\cos\psi_1}{\cos(\psi_1-\psi_2)}=\frac{\sqrt{1+\tan^2\psi_2}}{1+\tan\psi_1\tan\psi_2}=\frac{\sqrt{1+\dfrac{a^2L_2^2}{r_2^2}}}{1+\dfrac{a^2L_1L_2}{r_1r_2}}$$
$$=\frac{r_1S}{r_1r_2+a^2L_1L_2}.$$

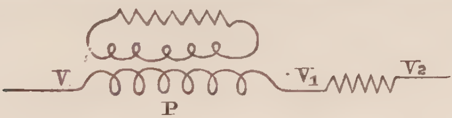
Hence
$$_m=\frac{\delta}{k}\;\frac{r_1S^2}{r_1r_2+a^2L_1L_2}.$$

Now for permanent currents,
$$\delta=k\frac{e}{r_2}C, \text{ and } eC=K\delta,$$
where K is the constant of the instrument for watts.

Therefore
$$k=\frac{r_2}{K},$$
or
$$p_m=K\delta\,.\frac{r_1(r_2^2+a^2L_2^2)}{r_2(r_1r_2+a^2L_1L_2)}.$$

In order to be able to make use of this result it is necessary to know the coefficients of self-induction of P + A and B ; the latter may be found once for all by any of the well-known methods and its value marked on the instrument ; but the former will require to be determined for the same values of the currents in the primary and secondary coils of the transformer as are flowing when the power is measured, since the apparent coefficient of self-induction of the primary coil depends on the saturation of the iron of the transformer and also on the current in the secondary coil. The best method of measuring L_1 under these conditions is that due to Joubert.

Connect an inductionless resistance R in series with the



primary, and pass an alternating current of known period through the two ; arrange the resistances of the primary and secondary circuits so that the currents in them have about the same values as when the power-measurement was made. Now connect a high-resistance Siemens electrodynamometer* between V and V_1, and let the reading be δ_1 ; again connect it between V_1 and V_2 and let the reading be δ_2.

Then

$$\frac{\sqrt{r_1^2 + a^2 L_1^2}}{R} = \sqrt{\frac{\delta_1}{\delta_2}},$$

$$L_1 = \frac{1}{a}\sqrt{R^2 \frac{\delta_1}{\delta_2} - r_1^2}.$$

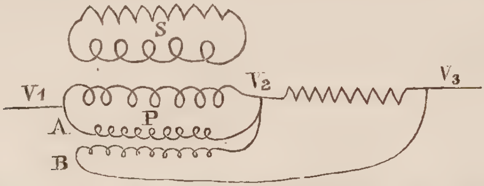
This method of calculating the mean power supplied to the primary of a transformer is, however, inconvenient, as it entails the use of a high-resistance dynamometer, with which two observations must be taken to obtain L_1, in addition to the trouble of adjusting the resistances of the two circuits to obtain the same conditions as when the wattmeter-reading was taken.

* A Cardew voltmeter may be employed, in which case

$$L_1 = \frac{1}{a}\sqrt{R^2 \frac{\delta_1^2}{\delta_2^2} - r_1^2}.$$

The following method, in which a high-resistance electro-dynamometer is employed, enables the power given to the primary coil to be measured from one reading without the knowledge of either the resistance or the coefficient of self-induction of the primary.

Let the two coils of the electrodynamometer be A and B, and let their resistances and coefficients of self-induction be $r_1 r_2$ and $l_1 l_2$ respectively; moreover, let $\frac{l_1}{r_1}=\frac{l_2}{r_2}$, that is to say, let the time-constants of the two coils be the same; this can be easily effected by putting an inductionless resistance in series with one or other of the coils. Connect as in the diagram.



Let the primary coil be put in series with an inductionless resistance R. Let the potential-difference between V_1 and V_2 $=E_1 \sin at$; then i, the current through the primary coil and through $R=\frac{E_2}{R}\sin(at-\psi)$, where E_2 is the maximum potential-difference between V_2 and V_3.

Let ψ_1 and ψ_2 be the angles of lag of the coils A and B respectively, and let i_1 and i_2 be the currents through them at some instant t.

Then, if δ be the reading of the torsion-head,

$$\delta=\frac{k}{T}\int_0^T i_1 i_2\,dt,$$

$$=\frac{k}{T}\cdot\frac{E_1E_2}{S_1S_2}\int_0^T \sin(at-\psi_1)\sin(at-\psi-\psi_2)\,dt,$$

$$=\frac{k}{2}\cdot\frac{E_1E_2}{S_1S_2}\cos(\psi_1-\psi_2-\psi);$$

where

$$S_1=\sqrt{r_1^2+a^2l_1^2} \quad \text{and} \quad S_2=\sqrt{r_2^2+a^2l_2^2}.$$

Now

$$\tan\psi_1=\frac{al_1}{r_1}, \quad \text{and} \quad \tan\psi_2=\frac{al_2}{r_2}.$$

Hence
$$\tan\psi_1 = \tan\psi_2, \text{ since } \frac{l_1}{r_1} = \frac{l_2}{r_2},$$
or
$$\psi_1 - \psi_2 = 0.$$
Therefore
$$\delta = \frac{k}{2} \cdot \frac{E_1 E_2}{S_1 S_2} \cos\psi.$$
Now mean power given to the primary coil, or
$$p_m = \frac{E_1 E_2}{2R} \cos\psi,$$
$$= \frac{\delta}{k} \cdot \frac{S_1 S_2}{R}.$$
For permanent currents,
$$\delta = k\frac{E_1}{r_1} \cdot \frac{E_2}{r_2} = kE_1 C \cdot \frac{R}{r_1 r_2}.$$
Also
$$E_1 C = \frac{K}{R}\delta,$$
where $\frac{K}{R}$ is the constant for watts.

Hence
$$k = \frac{r_1 r_2}{K}.$$
Therefore
$$p_m = \frac{K}{R} \cdot \delta \cdot \frac{S_1 S_2}{r_1 r_2}.$$
$S_1\, S_2$ may be written
$$r_1 r_2 \sqrt{\left(1 + \frac{a^2 l_1^{\,2}}{r_1^{\,2}}\right)\left(1 + \frac{a^2 l_2^{\,2}}{r_2^{\,2}}\right)} = r_1 r_2 (1 + \tan^2\psi_1).$$
Hence
$$p_m = \frac{K}{R} \cdot \delta \cdot (1 + \tan^2\psi_1).$$

In order that the non-inductive resistance R shall not absorb too much power, B should be the movable coil of the electro-dynamometer, as this is generally the one of lower resistance; $\frac{E_1}{S_1}$ should have about the same value as $\frac{E_2}{S_2}$, or
$$\frac{E_2}{E_1} = \frac{S_2}{S_1} \text{ approximately.}$$
Now
$$\frac{S_2}{S_1} = \frac{r_2\sqrt{1 + \tan^2\psi_2}}{r_1\sqrt{1 + \tan^2\psi_1}} = \frac{r_2}{r_1};$$

that is, if r_2 is considerably lower than r_1, E_2 will be in the same proportion lower than E_1, and hence R may be a good deal smaller than the impedance of the primary.

XXX. *The Magnetic Circuit of Dynamo Machines.*

By W. E. Ayrton *and* John Perry.*

[Plate VIII.]

In this paper we shall use the following symbols :—Dimensions are given in centimetres, current in amperes, potential differences and electromotive forces in volts, resistances in ohms.

We shall speak indifferently of the drum or Hefner-Alteneck and the cylindric Gramme armature—that is, the Gramme armature which receives its induction through its cylindric surface from two pole-pieces. It is easy to make the slight changes in the formulæ required for flat gramme ring-armatures like those of the Victoria, Gulcher, and Schückert machines, which receive induction through their flat sides, and for all machines with more than two pole-pieces.

r, outside radius of armature.

L, length of armature parallel to the axis.

k, a constant such that $kr\mathrm{L}=a_1$; k is as much as 1·25 in drum or Hefner-Alteneck armatures, and as little as 0·5 in short cylindric or other Gramme armatures.

t, the thickness of the winding of the armature.

d, the clearance between wire and pole-piece.

$\delta=d+t$, the total distance from iron of armature to pole-piece.

n, the revolutions per second of the armature.

v, the circumferential velocity of the armature in centimetres per second $v=2\pi rn$.

S, wires counting all round the outside of the armature.

A, amperes in each wire.

SA will be called the "ampere-wires" on the armature, being the ampere-turns if it is a Gramme, and being twice the ampere-turns if it is a Hefner-Alteneck.

α, amperes per sq. centim. flowing in the section of the armature-winding made by a plane at right angles to the axis.

α_1, the highest permanent value of α allowable.

* Read March 10, 1888.

ρ, the electric resistance of the armature-winding per cubic centimetre. That is, per centimetre of its length per sq. cm. of cross section. Of course ρ is greater than the specific resistance of copper, as the space is partly occupied by insulating material, and ρ is greater with finer wire; but for some practical purposes it is advisable to assume ρ constant for all kinds of winding.

$\rho\alpha^2$, watts developed as heat per cubic centimetre of the winding, so that total rate of development of heat $=2\pi rt\mathrm{L}\rho\alpha^2$.

a_1, cross section of iron of armature by a diametral plane.

a_2, area of pole-piece exposed to the armature, increased by a fringe of 0·8 δ in breadth all round.

a, the cross section of any other portion of the magnetic circuit which may be considered.

β, the induction in C.G.S. units, given in lines per square centimetre anywhere in the magnetic circuit.

β_1, the value of β in the iron of the armature.

β_1', the greatest value of β which it is convenient to employ in the armature when the machine is giving its greatest permanent output.

N, the total induction in the iron of the armature $\mathrm{N}=\beta_1 a_1$ or $\beta_1\, kr\mathrm{L}$.

$\frac{2\delta}{a_2}$ or $\frac{2\,(t+d)}{a_2}$ will be called the magnetic air-resistance, a not very incorrect or misleading expression.

$\frac{l\nu}{a\mu}$ will be called the magnetic resistance of a portion of the iron part of the magnetic circuit whose length is l, whose cross section is a, the permeability being μ, $\mathrm{N}\nu$ being the total induction there.

The whole iron magnetic resistance may be written $\Sigma\frac{l\nu}{a\mu}$.

This is for only one of the magnetic circuits if the machine has more than one.

In a well-designed modern machine there is no throttling of the induction anywhere, and $\frac{l}{a_1\mu}$ may be taken as the whole magnetic iron resistance, μ being the permeability of the iron of the armature, l the average

total length of a line of induction in the iron of the whole machine.

W watts, the power developed by the rotation of the armature.

W′, the permanent highest power developed.

$E\theta$, the rate of loss of heat in watts per square centim. of the cylindric outer surface of the armature. This is usually taken to be 0·2 in modern machines furnished with some ventilating arrangement. The value of $\sqrt{\frac{E\theta}{\rho}}$ may, we find, be practically taken as 288 in such machines; we shall call this constant q, as machines made by different makers differ greatly in its value, even where there are the same methods of ventilating and the same sizes of wires. With finer wires ρ is greater, as the space is less occupied by copper, so that q is less. If two thirds of the space be taken as occupied by copper, $q=288$ if $E\theta=0{\cdot}2$. In some armatures, which are always perhaps too safe from heating, we have found q to be as little as 150. In some cases it is considerably more than 300.

S_2, spires on the coils of the field-magnet part of one magnetic circuit. In the Edison-Hopkinson and Kapp machines there is only one magnetic circuit, and S′ is here the total number of spires on the field-magnets; but in the Manchester, Crompton, A-Gramme, and other forms there are two magnetic circuits, and S′ is here the number of spires on only one of the magnetic circuits.

A_2, amperes in each spire of the field magnet-coils; or

α', amperes per square centimetre of cross section of the winding.

Any person who has engaged in making measuring-instruments or dynamo machines is aware that for a given volume of winding, whether the wires are small or large, for the same distribution of temperature there will be the same number of ampere-turns and there will be the same rate of loss of energy by heating, if the volume of insulating material is always in the same ratio to the volume of the copper.

With fine wire the volume of insulating material becomes greater, but not to such an extent as to make useless this very important roughly correct practical rule for the makers of instruments and dynamo machines. This rule we have regularly used since 1881 in our measuring-instruments.

This has led us to speak of α the current in amperes per square centimetre of cross section of a coil, rather than the density of current in the copper alone, and we have been led to some general rulés of considerable practical interest in consequence.

To make our rules more complete, we begin with one which is well known. It will be observed that we use Mr. Kapp's method of counting wires on the armature, so as to make the rule suitable both for the Hefner-Alteneck and the Gramme armature.

$$\text{Total E.M.F. of armature} = \frac{n\mathrm{SN}}{10^8} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\left.\begin{array}{l}\text{E.M.F. developed in unit length}\\ \text{of wire passing through the field}\end{array}\right\} = \frac{vk\beta_1}{\pi 10} \quad . \quad . \quad (2)$$

Observe that it is only the wire on the convex outer surface of the armature which is here considered.

$$\mathrm{W} = \frac{2n\mathrm{N}}{10^8}\mathrm{SA}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\mathrm{W} = \frac{2v\mathrm{N}t\alpha}{10^8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The heat generated in the armature-winding per cubic centim. $= \rho\alpha^2$. The possible ventilation arrangements are better in Gramme than in Hefner-Alteneck armatures, and it is usual, therefore, to consider only the heat generated in the wires on the outside convex surface of the iron of either. This is $2\pi rt\mathrm{L}\alpha_1{}^2\rho$; and as the heat emitted is found, for the highest temperature θ at which it is considered safe permanently to work the machine, to be proportional to the convex surface $2\pi r\mathrm{L}$, if E is the rate of loss of heat in watts per square centimetre of this surface,

$$2\pi rt\mathrm{L}\rho\alpha_1{}^2 = \mathrm{E}\theta 2\pi r\mathrm{L},$$

or

$$t\rho\alpha_1{}^2 = \mathrm{E}\theta.$$

We find that $\dfrac{\mathrm{E}\theta}{\rho}$ may be taken as 83000 in the best modern

machines ; we shall call this q^2, so that

$$t\alpha_1^2 = q^2. \qquad (5).$$

We can, therefore, from (4), express the greatest permanent output of a machine W′, either in terms of α_1 or of t :

$$W' = 0{\cdot}00166 \frac{vN'}{\alpha_1}, \qquad (6)$$

$$W' = \frac{2q}{10^8} vN'\sqrt{t}, \qquad (7)$$

$$W' = \frac{2q}{10^8} vkrL\beta_1'\sqrt{t}, \qquad (8)$$

N′ and β_1' being the highest allowable values of N and β_1.

It is on the combination of (5) and (4) that we have based an important generalization regarding the magnetic circuit of the dynamo, which gives the title to our paper. Members of the Society are aware of the methods adopted by Mr. Bosanquet, Dr. Hopkinson, and Mr. Kapp in dealing with a magnetic circuit. Consider a closed tube of small cross section everywhere, passing through the iron of the armature, the air-spaces, the limbs of the field-magnet, and the yoke. Let the section vary so that the total induction is everywhere the same. Then, in a short length of the tube l, the line-integral of the magnetic force required to produce this induction is

$$\frac{l}{\mu}\beta,$$

where μ, the magnetic permeability of the material at any place, is 1 for air, and where its value for iron is given in the accompanying figure for various values of β. The curve B, Pl. VIII. fig. 1, is that given by Mr. Bidwell in his paper (Proc. Royal Society, No. 245, 1886). The curve H_1 has been obtained from Dr. Hopkinson's experiments on wrought iron in the Philosophical Transactions, 1885, plate 47. We wish that Dr. Hopkinson had in this paper given his experimental numbers, instead of plotting them in curves to such scales that one feels very great uneasiness in making measurements. We have determined the curve H_1 by taking a medium curve between Dr. Hopkinson's curves for rising and falling magnetizations. It is quite probable that, in view of the great differences in the behaviour of the same and

different kinds of iron, under different circumstances, Dr. Hopkinson did not feel that he was justified in drawing his curves on paper too finely divided. It will be observed that from $\beta=10000$ to $\beta=16300$ we may take $\mu=5221-0{\cdot}3071\beta$. Unfortunately for easy methods of calculation in many machines much larger values of β than 16300 are often used.

Curve H_2 is computed from Dr. Hopkinson's results for cast iron, and it shows how very important it is, when cast iron is used in the magnetic circuit of a machine, to have its section much greater than that of the wrought iron portions of the circuit, if throttling the induction is to be avoided.

Curves K_1, K_2, and K_3 are computed from the formulæ given in Mr. Kapp's paper for the wrought iron used in the armature, field-magnets, and yoke respectively of a dynamo machine.

The differences among these curves throw great light upon the fact that it is really impossible to predetermine the "*characteristic*" of a dynamo machine. Mr. Kapp gets over discrepancies by calculating a value of his leakage-resistance which suits the actual observations made on an already constructed dynamo machine; and Dr. Hopkinson's curves, although calculated on the basis of actual measurements of the leakage made on an already constructed dynamo, and although only computed for a very small portion of that part of the characteristic where the iron magnetic resistance is important (the only portion about which there is any difficulty), represent this small portion very indifferently indeed. Methods of calculating the leakage which have been put forward we can in no way believe in, for reasons given by one of us in the discussion on Mr. Kapp's paper at the Society of Telegraph Engineers. We are not in any way detracting from the merits of Dr. Hopkinson's beautiful theory. We are too well aware of the great services he has done us and of the enormous change which he has produced in the construction of dynamos.

Now the line-integral of the magnetic force round a closed magnetic circuit—called by Mr. Bosanquet the Magnetomotive Force—is the number of ampere-turns on the coils through which it threads it way $\times \frac{4\pi}{10}$. Hence if S_2A_2 be the

ampere-turns on the field magnet,

$$\frac{4\pi}{10}S_2A_2 = 2\delta\beta + \Sigma\frac{l}{\mu}\beta, \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

δ being $d+t$ the distance from iron of armature to iron of field-magnet, and of course the length of the tube in air is twice this distance as it goes into and out of the armature.

If N is the total induction through the iron of the armature, $\frac{N}{a_1}=\beta$ there and $\frac{N}{a_2}=\beta$ in the air-space.

If νN is the total induction through the iron of the field-magnet anywhere, then $\frac{\nu N}{a}=\beta$ there, so that (9) may be written

$$\frac{4\pi}{10}S_2A_2 = 2(d+t)\frac{N}{a_2} + \Sigma\frac{l\nu N}{a\mu},$$

or

$$N = \frac{\frac{4\pi}{10}S_2A_2}{\frac{2(d+t)}{a_2} + \Sigma\frac{l\nu}{a\mu}}; \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

and as N is the induction through the armature produced by a magneto-motive force $\frac{4\pi}{10}$ S_2A_2, equation 10 leads to $\frac{2(d+t)}{a^2}$ being called by analogy with other physical resistances, the air magnetic resistance, and $\Sigma\frac{l\nu}{a\mu}$ the iron magnetic resistance of the circuit. The members of the Society will perhaps allow us to put in this way Dr. Hopkinson's theory, although he himself may object to some of the terms we use.

Now let it be assumed that there is a value of β for the armature iron which it is best to use in all machines when giving their permanent output. ν may also be taken as practically constant. We are aware that to both these assumptions exception may be taken, but this will not be found to affect the practical general result which we arrive at. Then μ is known, so that if the lengths and cross sections of the iron everywhere are known, the iron magnetic resistance $\Sigma\frac{l\nu}{a\mu}$ is known. Inserting this value of N as N′ in (7) we

find

$$W' = \frac{2q}{10^8} v \sqrt{t} \frac{\frac{4\pi}{10} S_2 A_2}{\frac{2(d+t)}{a_2} + \Sigma \frac{lv}{a\mu}}; \quad . \quad . \quad (11)$$

and it will be found that this is a maximum for different values of t when

$$\frac{2t}{a_2} = \frac{2d}{a_2} + \Sigma \frac{lv}{a\mu}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

That is, when the magnetic resistance of the space occupied by the winding of the armature is equal to the resistance of the rest of the magnetic circuit.

When this is the case (11) reduces to

$$W' = \frac{2\pi q}{10^8} \frac{va_2}{\sqrt{t}} S_2 A_2. \quad . \quad . \quad . \quad . \quad (13)$$

That the ampere-turns on the field-magnet S_2A_2 may really produce the given induction β_1 in the armature, it is necessary that

$$S_2 A_2 = \frac{10}{\pi} \frac{a_1 t}{a_2} \beta_1;$$

and if we insert this value in (13) we have of course (7) again.

It is now the custom in making dynamos to let β be nearly constant in all parts of the circuit where the iron is the same, and to have $\frac{\beta}{\mu}$ as nearly constant as possible, if different kinds of iron are employed. If $\frac{\beta}{\mu}$ is larger anywhere than its average value for the rest of the circuit, we say that the induction is there "throttled," and throttling is only allowable in the armature, if it is allowable anywhere. If $\frac{\beta}{\mu}$ is nearly constant everywhere in the iron, we may take $\frac{l}{a_1\mu}$ as the total iron resistance of the circuit, l being the average length of the lines of magnetic induction in the iron of the whole circuit, and then we may use

$$\frac{l}{a_1\mu} \text{ instead of } \Sigma \frac{lv}{a\mu}$$

in (10), (11), and (12); μ being the permeability of the armature iron when the induction is β_1'.

In the above investigation, the iron resistance is supposed to be given, and also the exciting power in ampere-turns. At present this seems a sufficiently practical basis for the calculation, as we usually fix first the size of armature, then arrange as short a magnetic circuit as possible, which must leave sufficient room for the exciting coils. We see, however, that a larger generalization is possible when we know with certainty what is the limiting thickness of winding on the field-magnets. We were allowed to assume in the armature winding that $t\alpha_1^2$ is constant, α_1 being the greatest permanent current per square centimetre allowable. When the winding is thin, so that the temperature is nearly uniform in the winding, and only when this is the case, is such a rule allowable, and it is not allowable in cases where the winding is of the thickness usual in field-magnet coils. We are at present experimenting on this subject, but there are considerable difficulties in the way of obtaining practical rules. We have no doubt, however, that there is such a rule as

$$S_2'A_2'=p\lambda.$$

That is, the greatest number of ampere-turns which can usefully and permanently be applied on a field-magnet coil of length λ is proportional to λ. If l is given, the configuration of the machine enables λ and therefore $S_2'A_2'$ to be fixed. Then $S_2'A_2' \div \frac{2l}{\mu_1} = \beta_1$ enables β_1 and μ_1 to be calculated by trial if there is sufficient information about the character of the iron. $\frac{2t}{a_2}$ is now made equal to $\frac{2d}{a_2} + \frac{l}{a_1\mu_1}$, so that the important dimensions are fixed. Should β_1 be great, considerations of cost of the field-magnet winding may come in, to alter completely the design of the machine, but for a given configuration of machine this is the practical method of working.

From experiments, the results of which were published before this Society on March 12th, 1887, we came to the conclusion that there is a definite resistance at a joint in a magnetic circuit; and Professor Ewing found that by cutting a bar into two, four, and eight pieces, the magnetic permeability seemed to alter from 1220 to 980, 640, and 480 respectively, the joints being tooled up in the usual way. When the joints are carefully scraped he also found that

they materially increased the magnetic resistance unless considerable pressure was applied. Now as a stress of less than one quarter of the ordinary stresses to which wrought iron is subjected in machinery materially diminishes the permeability of iron, it is obvious that all joints in the magnetic circuit ought, as much as possible, to be got rid of, consistently with easy manufacture; or, when they exist, they ought to be made of as large area as possible, by letting one piece into another to a considerable depth.

In the design of a machine, the question of possible leakage is of considerable importance. It will often be found that in the effort to make l short and a_2 large, machines are designed with enormous amounts of leakage not only from one pole to the other, but from either pole to the middle of a neighbouring field-magnet coil.

One of the best ways of finding the nature of the probable magnetic leakage in a dynamo, before it is constructed, is to construct a small model of the same kind of iron, exciting the field-magnets, and exploring by means of a ballistic galvanometer. Another simpler way, which gives a considerable amount of information, and which we have employed, is to make a model of wood, covering certain judiciously selected parts, such as the poles and armature, and half the field-magnets with metal, immerse the model in a barrel of rain-water, and find the electric resistance between one part and another when electric potential-differences are established between them. On account of the ease with which the model may be rearranged in configuration, this method of working gives interesting results; but these results, when applied in the magnetic case, must be used judiciously and with the knowledge that the permeability of iron is not a fixed quantity.

It is obvious that the best section of a field-magnet limb is the circular, but considerations of possible leakage to the middle of the limb from the armature or a pole-piece, and other considerations relating to the configuration of the machine, often cause us to give to the section a rectangular or oval section.

The Characteristic of a Dynamo.

In a letter written by one of us from Japan, in January

1879*, before he had seen a dynamo, the necessity was shown for establishing an algebraic relation between the E.M.F. developed in the armature and the current exciting the field-magnets. It was pointed out that E the E.M.F. in the armature was proportional to the field and speed, and that this led to

$$E = p + qc + \Sigma \frac{ac}{b+c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

for any given speed. It was also pointed out that a tangent function might also be employed, but that it would not lend itself so readily to calculation. We had, both, in 1878 used a curve to express this relationship, but it was not until 1881, when we met M. Deprez, and learnt of his work, that we had any conception of the many calculations which might be made by graphical methods, using the curve as a fundamental relation. Herr Fröhlich uses one term of the above expression:—

$$E = \frac{ac}{b+c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

He regards p the permanent field as 0, and he has no term which remains proportional to the current. We shall speak of (15) as the Fröhlich formula, because it usually goes under that name. It is, of course, an empirical formula, like (14).

The reasoning which led us to regard (14) as a *rational* formula, was based on a magnetic theory which need not here be expounded.

A very great necessity exists for having some such empirical formula as (15), but, as is well known, it is quite incorrect when c is small. It is not nearly so incorrect as we might imagine it to be from the statements made by Mr. Kapp (p. 529, Journ. Soc. Tel. Eng. and Elect. vol. xv. 1887), who has given in his fig. 3 a most absurdly unsuitable Fröhlich curve, or by Dr. Hopkinson (plate xvi. Phil. Trans. 1886), whose Fröhlich is also not the most suitable. It is obvious that the Fröhlich ought not to be expected to agree with the real characteristic near the origin. It ought to be made to agree most perfectly with the working part of the characteristic. To effect this purpose let the observed values

* Not published till 1885, and then in a somewhat mutilated form in the 'Electrician' of November 20.

of $\frac{E}{C}$ and E be plotted as the coordinates of points on squared paper. The straight line which lies most evenly among the points for the working values of E satisfies the equation

$$\frac{E}{C}b+E=a.$$

The measured coordinates of two points on this straight line enable b and a to be calculated.

Thus, for example,

$$N=\frac{1364\,F}{1+2{\cdot}7\times10^{-4}\,F}$$

will be found to satisfy the observations published by the Drs. Hopkinson for the Manchester dynamo from F=6000 to F=30,000 with a wonderful amount of accuracy. But for the early part of the curve it is quite unsuitable. F is $\frac{4\pi}{10}\times$ ampere-turns in one coil of the machine, or the magneto-motive force.

The uses made by Herr Fröhlich and Prof. Rücker of (15) in a theory of compounding show how important it is to have a simple empirical formula. Indeed in our patent specification of 1882 we base the theory of compounding on the simplest of all empirical formulæ,

$$E=aC. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

And it is by means of this formula that the theory can be best put before students ; if our easy reasoning is properly grasped, the practical electrical engineer will be able to use the results graphically with actual characteristics in the manner employed by the most experienced men at the present day.

Instead of speaking of E the E.M.F. produced in the armature, which is proportional to N the magnetic field and n the speed, as shown in (1) it is preferable to speak of N the field itself. And instead of referring merely to the current exciting the field-magnets, it is preferable to speak of the ampere-turns S_2A_2, or, better, of the magneto-motive force in C.G.S. units, $\frac{4\pi}{10}S_2A_2$.

We have tried in vain, during perhaps two months of very hard work, to express the two constants a and b of (15) in

terms of the dimensions of a dynamo machine. We now believe that this cannot be effected. We have already given the reason why the linear law connecting the β and μ cannot be used in obtaining a formula. In any case such a formula would be very different from that of Fröhlich. But in view of the important relation (12) which we have established, that when the machine is giving its best permanent output,—

The air magnetic resistance of the space occupied by winding on the armature = all other magnetic resistance,

it is not difficult to arrive at practical rules of working. It will be noticed that in existing machines we may, in general calculations, neglect the resistance of the clearance-space d.

The most important fact to be kept in mind by constructors of dynamos is, that the magnetic air-resistance is the governing factor. Until half the highest induction is reached we may neglect altogether the resistance of the iron; and indeed for many calculations it is sufficient to take

$$N=\frac{4\pi}{10}\frac{a_2}{2\delta}S_2A_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where a_2 is the area of the pole-piece exposed to the armature (increased by the fringe-area, which Dr. Hopkinson estimates to be of a breadth $0{\cdot}8\,\delta$), and δ is the distance from iron of armature to iron of pole-piece.

(17) gives, in fact, the straight part of the characteristic. In well-constructed dynamos the magnetic iron resistance is equal to the air resistance $\frac{2\delta}{a_2}$ when the machine is working at its most permanent output; and it is only necessary to know β', the induction convenient to use when this is the case, to be able to calculate N and S_2A_2 for a point on the curved part of the characteristic. Thus

$$a_1\beta_1=N_1=\frac{4\pi}{10}\frac{a_2}{4\delta}S_2A_2;$$

so that if a_1 and β_1 are known, S_2A_2 can be found. Thus in the Manchester dynamo described by Dr. Hopkinson (Trans. Roy. Soc. 1886),

$a_1=220{\cdot}5$ square centim.,
$a_2=839{\cdot}5$ „ „
$\delta=\ \ 0{\cdot}8$ centim.

Taking $\beta_1 = 18{,}460$ lines per square centim., we find

$$a_1\beta_1 = 4\cdot 07 \times 10^6.$$

Now

$$\frac{4\pi}{10}\frac{a_2}{2\delta} = 660.$$

So that $N = 660\ S_2A_2$ represents the straight part of the characteristic.

Again,
$$N_1 = a_1\beta_1 = 4\cdot 07 \times 10^6,$$
and
$$a_1\beta_1 = 330\ S_2A_2,$$
or
$$S_2A_2 = \frac{4\cdot 07 \times 10^6}{330} = 12330.$$

Plotting $N = 4\cdot 07 \times 10^6$ and $S_2A_2 = 12330$ as the coordinates of a point on squared paper, and plotting also the straight line from the origin $N = 660\ S_2A_2$, a man who has seen characteristics before will be able to draw the characteristic of this machine with a fair amount of accuracy, especially if he recollects that the iron is near saturation. Those, however, who have less experience may find the Fröhlich, which for any dynamo passes through the point on the straight line which represents $N = 10{,}000\ a_1$, and the second point just found, rounding off the straight line into the Fröhlich by hand. In applying our rule to the Manchester dynamo we employed neither of these methods, as one's judgment is vitiated when one has seen the actual observations; and we have assumed that the very roughly correct rule,

$$N = p \tan^{-1}(h\,.\,S_2A_2),$$

may be applied, making its slope the same as that of the straight line at the origin, and making it pass through the point whose position we have calculated. In this case

$$ph = 660;$$

and it will be found that

$$p = 34\cdot 65 \times 10^5,$$

and

$$h = 19\cdot 03 \times 10^{-5}$$

for this case.

As Dr. Hopkinson has given his characteristics, not in terms of ampere-turns S_2A_2, but in absolute units, calling

$\frac{4\pi}{10} S_2 A_2$ by the letter F to denote the magneto-motive force in absolute units, the above formula becomes for this comparison,

$$N = 34{\cdot}65 \times 10^5 \tan^{-1} (15{\cdot}15 \times 10^{-5} F).$$

In the figure we have plotted the curve obtained by calculation as A; B shows Dr. Hopkinson's calculated curves; the dotted line passes as nearly as possible through the points determined by experiment. It will be observed that the Drs. Hopkinson have only ventured to calculate the characteristic up to $F = 12{,}000$ on the ascending, and 9500 on the descending part; whereas the observations extend to $F = 29{,}500$. As the very straight part near the origin presents no difficulty, it may be said that all their elaborate calculation represents only the characteristic from about $F = 6000$ to $F = 12{,}000$, and that part very indifferently. It is, besides, to be observed that in an actual dynamo there are no such differences between observations made with steadily increasing and with steadily diminishing magneto-motive forces, as the theory requires.

In the above calculation we have neglected the clearance d, as we did not know it. It would have been easy to calculate a value of d, possibly not very different from the real value, to make our curve agree more closely with the observations. In conclusion we would say that, in our opinion, it is impossible, until a machine is constructed, to compute its characteristic with sufficient accuracy for such purposes as the determination of coils in compound winding. But the probable characteristic may be determined in the way here described, with sufficient accuracy for a number of useful purposes. It is based on the facts:—(1) When a machine is working at its best permanent output, its iron magnetic resistance plus the air magnetic resistance of the clearance is equal to the air magnetic resistance of the space on the outside of the armature occupied by winding. (2) At the beginning the air-resistance is alone of any importance.

XXXI. *The Variation of the Coefficients of Induction.* By

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[Plate IX.]

1. THERE are three ways of defining the coefficient of self-induction, which lead to the same result if the magnetic permeability of the medium is a constant quantity, but which lead to three different results if it is a variable one, as in the case of iron.

The coefficient of self-induction L of a coil of wire through which a current C is passing may be defined as the ratio between the back electromotive force and the time-rate of change of the current C to which it is due; or it may be defined as the ratio between the flux of induction through the coil and the current producing it; or it may be defined with reference to the electrokinetic energy possessed by the current C.

The three definitions are expressed by the equations

$$(1) \qquad e = L_1 \frac{dC}{dt},$$

$$(2) \qquad N = L_2 C \quad \text{and} \quad e = \frac{d(L_2 C)}{dt},$$

$$(3) \qquad T = \tfrac{1}{2} L_3 C^2.$$

In these equations e is the back electromotive force produced by varying the current C, N is the number of lines of force passing through the coil, T is its kinetic energy, and L its coefficient of self-induction.

2. If the magnetic medium is air, L_1, L_2, and L_3 are identical. If the medium be wholly or partially composed of iron, this is no longer the case. The values of L differ from each other, and vary with C. The value of L_1 can easily be found in terms of L_2 from the equation

$$L_1 = L_2 + C \frac{dL_2}{dC},$$

which is an immediate consequence of the first two equations of definition.

* Read April 14, 1888.

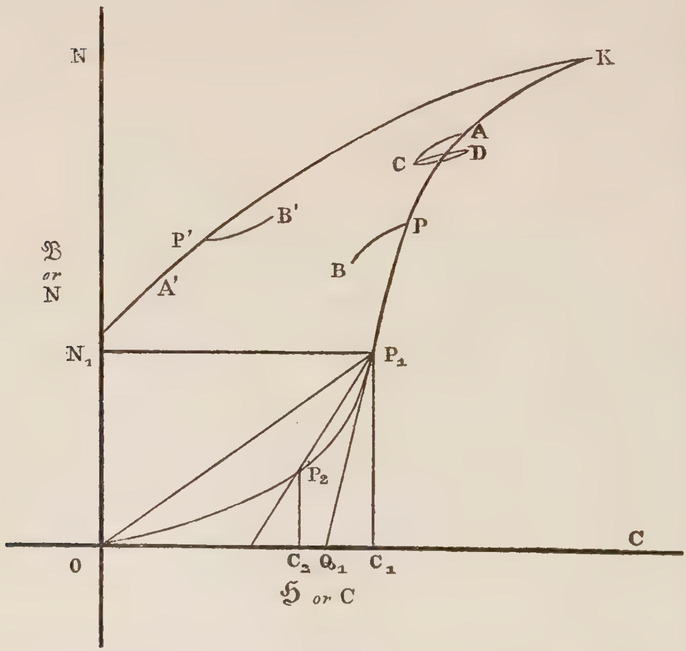
We see that $L_1 = L_2$ only when $C=0$ or when $\frac{dL_2}{dC}=0$, and that L_1 is greater than L_2 for small currents where L_2 is increasing with C.

Since $\mathfrak{H}$, the magnetizing force, is proportional to C; and since N is a measure of $\mathfrak{B}$, the average value of the flux of induction per unit area, we see that L_2 is directly proportional to μ, the average value of the coefficient of magnetic permeability for the magnetizing force represented by the current C.

If, therefore, the medium be wholly of one kind, L_2 will be a measure of the permeability of that medium. In any case, however, if we know the relation connecting $\mathfrak{B}$ with $\mathfrak{H}$, we shall be able to determine the way in which the coefficients vary with the magnetizing force.

If OPKP′ (see fig. 1) be the curve connecting $\mathfrak{B}$ with $\mathfrak{H}$

Fig. 1.



(or N with C), the value of L_2 for any point P_1 on it corresponding with the current OC_1 and the flux of induction ON_1

will be represented by the tangent of the angle which the line OP_1 makes with the line OC_1.

Since

$$L_1 \frac{dC}{dt} = e = \frac{d(L_2 C)}{dt} = \frac{dN}{dt},$$

we have

$$L_1 = \frac{dN}{dC} \propto \frac{d\mathfrak{B}}{d\mathfrak{H}},$$

and L_1 will consequently be represented by the tangent of the angle $P_1Q_1C_1$ which the tangent to the curve at P_1 makes with the line OC_1.

Moreover,

$$L_3 = \frac{2\int C\,dN}{C^2} \propto \frac{\int \mathfrak{H}\,d\mathfrak{B}}{\mathfrak{H}^2},$$

and will be represented by the ratio of the area enclosed by the lines ON_1, P_1N_1, and the curve OP_1 to the square on the line OC_1.

3. Maxwell's method of determining the coefficient gives L_2, because the quantity of electricity discharged through the galvanometer is proportional to the number of lines of force inserted by the establishment of the current C (or to the number removed by stopping the current), while the steady deflection due to a small derangement is proportional to C directly. The method therefore gives the ratio of N/C or L_2. Professor Ayrton has generalized this method by altering the current from one value to another, instead of establishing it from zero to its full value. The quantity discharged through the galvanometer is proportional in this case to $N_1 - N_2$, where N_1 is the number of lines of force corresponding with the first current C_1, and N_2 that corresponding with the other current C_2. If the steady deflection caused by a derangement from balance be taken when a current C is flowing through the coil, Maxwell's formula will give the ratio of $\frac{N_1 - N_2}{C}$; and if we multiply this by $\frac{C}{C_1 - C_2}$, we shall obtain the value of $\frac{N_1 - N_2}{C_1 - C_2}$, or the tangent of the angle which the chord P_1P_2 makes with the line OC.

By observing the ratio $\frac{C_1 - C_2}{C}$, in addition to the ordinary

quantities measured in Maxwell's method, it is possible to determine in this way the rate of slope of the curve of magnetization. If $(C_1 - C_2)$ is very small compared with $\frac{1}{2}(C_1 + C_2)$, the value obtained by Prof. Ayrton's method is L_1; but the result is practically the same if the curve of magnetization does not bend very much between the values of OC corresponding with C_1 and C_2.

Coefficients for small Currents.

4. It follows from the shape of the initial part of the curve of magnetization that the value of L_2 must at first increase with the current. The following experiments show that this is the case. A bar of best Swedish iron, 14 in. long by $\frac{1}{2}$ in. in diameter, was bent into the shape of a horseshoe. Each end was surrounded by a bobbin, $2\frac{1}{2}$ in. in diameter and $4\frac{1}{2}$ in. long, wound with 400 turns of wire. The self-induction of the two bobbins in series was obtained by Maxwell's method. The battery used was an ordinary Leclanché cell. The swings were in most cases taken on breaking the circuit, since, if the galvanometer-key be pressed just before the battery-key is released, the error caused by a slight want of balance in the bridge is reduced to a minimum. For a similar reason, the deflection corresponding with a known derangement was always estimated from the difference between two deflections. The current flowing through the coil was calculated from the electromotive force of the battery and the resistances in the network. The following numbers represent the mean of several concordant values :—

A.	L_2.	$L_2 - {\cdot}2A$.
·047	·0514	·0420
·056	·0539	·0428
·060	·0549	·0429
·065	·0554	·0424
·068	·0564	·0428
·079	·0577	·0423
·091	·0609	·0427
·107	·0634	·0420

In this table L_2 is the coefficient of self-induction of the coil in secohms, and A is the current in amperes flowing through it. The experiments are in fair accordance with the

relation

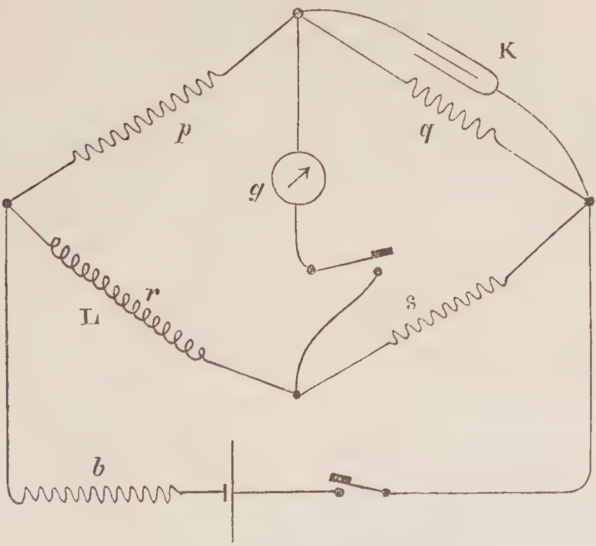
$$L_2 = \cdot 2A + \cdot 0425.$$

Since A is proportional to $\mathfrak{H}$, and L_2 is proportional to $\mathfrak{B}/\mathfrak{H}$, the above experiments tend to show that the part of the curve of magnetization between the points corresponding with the currents ·047 and ·107 ampere is a parabola of the form

$$\mathfrak{B} = a\mathfrak{H}^2 + b\mathfrak{H}.$$

5. It was, however, desirable to see whether this relation would still hold good for smaller magnetizing forces. With this view the experiments were repeated some months afterwards in a somewhat different way. The coefficient was determined by comparing it with the capacity of a standard condenser. Two arms, p, q, of a Wheatstone bridge (see fig. 2) consisted of doubly wound resistance-coils of 10,000 ohms each. The resistance g of the galvanometer used was also about

Fig. 2.



10,000 ohms. To the coil q was shunted a condenser of $\frac{1}{3}$ microfarad capacity. The opposite branch r of the bridge contained the electromagnet whose self-induction was required. The resistance of the arm s was 5 ohms, and that of r was adjusted till there was balance. The battery used was

an accumulator. The currents were changed by inserting more or less resistance into the battery-circuit. If the bridge is balanced for steady currents, and the battery-circuit be opened, the swing θ_1 produced will be proportional to $L_2 - Kps$, where L_2 is the coefficient required and K is the known capacity. If one of the terminals of the condenser be now disconnected from the bridge, and a second swing θ_2 be taken on breaking circuit, we have

$$\frac{\theta_2}{\theta_1} = \frac{L_2}{L_2 - Kps},$$

or

$$L_2 = \frac{\theta_2}{\theta_2 - \theta_1} Kps.$$

This method, although only comparative, has several advantages over Maxwell's absolute method. It is quicker and simpler ; there are only two quantities to observe, and the readings may be taken immediately after each other. It is, moreover, not so necessary to have a good ballistic galvanometer. It will generally be best to work with the relations

$$p = q, \quad p + r = 2g, \quad Kps = 2L.$$

The best ratio, r/p, will be determined by the resistances and batteries available, and by the currents it is desirable to use. In the following experiments these relations were not adhered to, because the galvanometer was sufficiently sensitive, and because it was desirable to alter the currents flowing without rendering it necessary to readjust the bridge.

In all swing methods it is necessary to have the bridge well balanced for steady currents. A fine adjustment can be conveniently obtained by sliding a bare wire of suitable thickness round a terminal, since it is not generally necessary to know the resistance of the arm of which it is a part, and it can of course be placed in the arm of unknown resistance. I believe the device used in the Cavendish laboratory is to shunt one resistance-box by another. The balance of the bridge has to be so good that the heating caused by the momentary passage of the current is often sufficient to destroy it.

The annexed table indicates the results obtained with the electromagnet with the iron core. The swings were obtained by breaking the battery-circuit.

b.	θ_2.	θ_1.	L_2.	A.
0	688	508	·0637	·220
1	627	462	·0634	·200
5	444	321	·0606	·147
10	367	261	·0577	·110
20	230	160	·0547	·073
30	164	111	·0517	·055
100	49·5	29·75	·0419	·020
130	38·0	22·7	·0414	·016
200	24	13·5	·0381	·0105
230	20·5	11·0	·0360	·0092

The values obtained for L_2 are slightly lower than the corresponding values obtained by Maxwell's method three months previously. This is probably due to the fact that the electromotive forces of the cells used were only approximately determined. The differences, however, are small, and may have been due to alterations in the state of the iron. The values for L_2 obtained for small currents are seen to be much lower than those given by the equation

$$L_2 = \cdot 2A + \cdot 0425.$$

They are, however, very much higher than that of the coil with the iron core removed. The coefficient in this case was ·0028 secohms.

6. The coefficient was very much increased by keeping a piece of soft iron pressed against the poles of the electromagnet. As, however, under these circumstances, a large fraction of the magnetism remained on removing the magnetizing current, the values obtained depended upon the previous history. Thus the value of L_2 obtained for a magnetizing current of about ·04 ampere varied between ·337 secohm and ·150 secohm, according as the flings were obtained on reversing the current, on breaking the battery-circuit, or on making it so as to include or exclude the permanent magnetism. The numbers obtained showed, moreover, that the curve of magnetization was not quite symmetrical with respect to positive and negative currents.

The following table gives the values of L_2 obtained for different currents by the method of reversals. The resistances and electromotive forces used to obtain the last five observations were quite different from those used to obtain the first six.

A.	L_2.	$\mathfrak{B}$.	μ.	$\mathfrak{H}$.
·0366	·184	665	780	0·852
·0275	·156	424	661	·640
·0220	·141	306	597	·512
·0147	·112	163	474	·342
·0110	·091	99	385	·256
·0055	·071	38·6	301	·128
·0029	·062	17·8	263	·068
·0044	·066	28·7	279	·102
·0100	·091	89·8	385	·233
·0200	·128	253	542	·465
·0400	·189	746	801	·931

7. As the magnetic circuit in this case was entirely composed of iron, it was easy to reduce the observations to absolute measure. The values of the induction $\mathfrak{B}$, the magnetic force $\mathfrak{H}$, and the magnetic permeability μ were calculated in C.G.S. units from the formulæ

$$\mathfrak{B} = \frac{10^8}{nS} L_2 A, \quad \mathfrak{H} = \frac{4\pi n}{10 l} A, \quad \mu = \frac{10^9 l}{4\pi n^2 S} L_2,$$

where A was the current in amperes, L_2 the coefficient of self-induction in secohms, S and l the mean values of the sectional area and length of the magnetic circuit in centimetres, and n the number of turns.

The iron core was half an inch in diameter and 14 inches long, and the distance between the poles was 3 inches. The number of turns was 800. Whence

$$l = 17 \times 2{\cdot}54 = 43{\cdot}2, \quad S = \frac{\pi}{4}\left(\frac{2{\cdot}54}{2}\right)^2 = 1{\cdot}27, \quad n = 800.$$

If these values are substituted, we obtain

$$\mathfrak{B} = 98{,}700\, L_2 A, \quad \mathfrak{H} = 23{\cdot}3\, A, \quad \mu = 4{,}240\, L_2.$$

The numbers obtained very approximately fulfil the following relations:—

$$L_2 = 0{\cdot}05 + 3{\cdot}9\, A,$$
$$\mu = 210 + 720\, \mathfrak{H},$$
$$\mathfrak{B} = 210\, \mathfrak{H} + 720\, \mathfrak{H}^2.$$

The values were obtained for reversals of magnetizing force whose semiamplitude $\mathfrak{H}$ was greater than 0·06 and less than 0·9 C.G.S. units. Beyond these limits the value of $\mathfrak{B}$ probably differs from that given by the above relation. By employing the secohmmeter of Professors Ayrton and Perry, it would have been possible to obtain values of L_2 for much

smaller magnetizing forces than those used above, and therefore for fields very much weaker than that due to the earth's magnetism.

Coefficients for large Currents.

8. The coefficient of self-induction of a coil with an iron magnetic circuit can far more easily be obtained for strong magnetic forces when the magnetism is maintained by an independent magnetic field than when the magnetism of the iron has to be excited by the current through the coil itself. In a transformer, for instance, where there are two coils identically situated with respect to the iron circuit, the coefficients of self and mutual induction are in the ratio

$$n_1^2 : n_2^2 : n_1 n_2,$$

where the ratio of the turns on the first coil to that on the second is $n_1 : n_2$. This will be true whatever the state of magnetization. If, therefore, one coil is used to excite the magnetism, and the other to test it, the values of the coefficients can be obtained without the least difficulty. Again, it is easy to obtain by Maxwell's method the value of L for the armature of a Series dynamo when the field-magnet coils are used independently to excite magnetism in the armature. Experiments were made in February 1887, at Prof. Ayrton's suggestion, to determine the coefficient of self-induction of the armature of a Gramme dynamo of the A pattern. The numbers obtained by Mr. S. Watney and the writer were as follow :—

Amperes round field-magnets.	Self-induction, in secohms.
0·0	·0218
6·1	·0179
15·1	·0135
24	·0122
29	·0117

9. When, however, it is desired to determine the self-induction of the field-magnet coils of a dynamo when a strong current is flowing through them, several difficulties present themselves. If strong currents are to be maintained the resistances must be small, unless a large amount of power is to be wasted and unless very high electromotive forces are procurable. On the

other hand, the self-induction to be measured is very large. The time-constant, which determines the rate at which the magnetism will change, will therefore be large; and unless a sufficiently ballistic galvanometer is to be had, it is not easy to obtain L in absolute measure.

At the suggestion of Prof. Ayrton some experiments were made at the commencement of 1887 to determine the variation of the self-induction of the field-magnet coils of a Ferranti dynamo. The method* used was the modification of Maxwell's method already alluded to. (The coefficient has been since compared with the capacity of a standard condenser by the swing method; the value obtained for a current of 0·01 ampere was 0·61 secohm, and for larger currents larger values were obtained.) When a strong current was flowing through the coils the resistance of the circuit was only a few ohms, and the coefficient was probably larger than 0·6. The time-constant was therefore generally larger than one tenth of a second, and the discharge could never be considered completed in less than a second. Probably no galvanometer would have been sufficiently ballistic under these circumstances to give good results in absolute measure. As, however, the only galvanometer conveniently situated with reference to the dynamos was one of the D'Arsonval type, with a short period and large logarithmic decrement even when on open circuit, the hope of any but comparative measurements was abandoned.

10. It is possible to calibrate any galvanometer for ballistic purposes by charging a condenser to a standard potential and discharging it through a resistance in series with the galvanometer to be calibrated. By suitably altering the resistance or the capacity the time-constant of discharge may be made to have any value. This was done with the D'Arsonval galvanometer, in order to interpret the results obtained. A condenser of 18·6 microfarads capacity was charged by a Latimer-Clark's cell, and then discharged through the galvanometer (whose resistance was 700 ohms) in series with a variable resistance, R ohms. The time-constant of discharge was

* This method, due to Prof. Ayrton, will be found more fully described, together with several other methods, in a paper by the present writer on "The Measurement of Self-Induction, Mutual Induction, and Capacity," Journ. Soc. Tel. Engineers, May 1887.

therefore

$$18{\cdot}6(\mathrm{R}+700)10^{-6} \text{ seconds,}$$

and the ratio of this to 1·66 was the ratio of the time-constant to the period of the galvanometer, and is denoted in the following table by the letter ρ. Observations were made of the value of the first swing of the galvanometer-needle for many different values of R. A few of these are given below.

R.	ρ.	First Swing (mean).
0	·008	293·5
1000	·019	291·2
5000	·064	269·0
10000	·120	234·0
50000	·568	105·5
100000	1·128	61·5
200000	2·248	36·0
300000	3·368	25·0

Thus, for the particular galvanometer tested, the throw produced by a given discharge when the time-constant is one tenth of the period is only about 80 per cent. of that produced by the instantaneous discharge of the same quantity; and when the time-constant is equal to the period, the throw is only about 20 per cent. of the corresponding throw for instantaneous discharge. Experiments since made with a high-resistance Thomson astatic galvanometer, with a period of 10 seconds, have yielded practically the same results. In the actual experiments on the Ferranti field-magnet coils, it would have been possible to interpret the meaning of the galvanometer-swings if the time-constant of discharge had had any fixed value, or if this value had been known; but the resistances and self-induction were continually varying, and the self-induction was always unknown. It would probably have been impossible even to compare the results with each other had it not been for the fact that, as the currents increased, the resistance and self-induction of the circuit diminished simultaneously, so that the time-constant of discharge tended to remain fixed in value.

11. The experiments on the dynamo coils were carried out by Messrs. Rossiter and Watney together with the writer.

The electromotive force used was obtained from accu-

mulators, and amounted to about 100 volts. The bridge was kept balanced for steady currents. The only resistance altered was that in the battery-circuit. The currents were changed by switching resistances into or out of the battery-circuit. If Q is the quantity of electricity discharged through the galvanometer when the current in the coil changes from C_1 to C_2,

$$L_1 = K \frac{Q}{C_1 - C_2},$$

where K is a function of resistances only, and is independent of the resistance of the battery branch when the bridge is balanced for steady currents. If the discharge Q produces a throw θ,

$$Q = k\theta,$$

where k depends, among other things, on the time-constant of discharge, and therefore varies with the self-induction and with the resistance in the battery branch. We may, however, regard k as being approximately constant, and we may accordingly take the values of $\frac{\theta}{C_1 - C_2}$ to represent the values of the coëfficient for the current $\frac{1}{2}(C_1 + C_2)$. In a series of experiments the currents were changed by successive steps from a small value to about 13 amperes, and decreased through the same stages in inverse order. The cycle was repeated many times, both for positive and for negative currents; and the numbers given in the following table are a fair sample of the results obtained. L_i and L_d are the values of $\theta/(C_1 - C_2)$ for increasing and decreasing currents respectively. A is the value in amperes of $\frac{1}{2}(C_1 + C_2)$.

A.	L_i.	L_d.
0·21	98	137
0·69	115	143
1·13	118	133
1·65	126	131
2·27	126	128
2·75	131	132
3·04	132	137
3·39	113	111
3·91	113	111
4·60	104	100
5·60	106	101
7·15	78	72
9·70	61	51·5

The numbers obtained, although not altogether satisfactory, leave no doubt about the way in which the coefficients change. For increasing currents L_i at first increases and then diminishes, while for decreasing currents L_d begins at a lower value than the corresponding value of L_i and continually increases as the current is decreased. This is exactly what is to be expected when it is remembered that the value of L is represented by the slope of the curve of magnetization. If the curve connecting $\mathfrak{B}$ with $\mathfrak{H}$ be like that indicated in fig. 1, the curve connecting $d\mathfrak{B}/d\mathfrak{H}$ or L with $\mathfrak{H}$ will be such that for increasing values of $\mathfrak{H}$ the ordinates L will at first increase and then diminish, while for decreasing values of $\mathfrak{H}$ the ordinates will continually increase. It is noteworthy that a discontinuity will occur at the cusp K, and the value of L will suddenly diminish as the change of current alters from an increase to a decrease, or *vice versâ*.

The general shape of the curve connecting L with $\mathfrak{H}$ will be the same whether the value of $\mathfrak{H}$ has been diminished to zero from positive or from negative values before the experiments are made. This follows from the fact that the slope at a point on a Ewing's cycle increases from each cusp to the point of inflexion. The coefficient L will be a maximum for those values of the magnetizing current C at which the points of inflexion occur. As the cycle only has one point of inflexion on each branch, the two parts of the curve connecting L with C will each have one maximum point, and one only. The two maximum values of L will not, however, occur at the same value of the current. They will occur at a small positive value of C for currents changing in the positive direction, and at a small negative value of C for currents changing in the negative direction. From this it follows that for increasing currents, L will at first increase and then diminish; while for currents decreasing to zero, L will continually increase. This statement must be modified for small cycles, which do not generally have points of inflexion. Here the coefficient will increase continually from one cusp to the other, and decrease discontinuously on rounding each cusp.

Experiments were also made on the field-magnets of a Gramme dynamo, the currents used varying in this case up to 30 amperes. The same kind of results were obtained.

12. Several remarkable effects were obtained, due to previous history of magnetization, which are easily explicable on reference to the researches of Professor Ewing*.

Suppose the current has been altered until a point P (see fig. 1) on the curve of magnetization is reached. The effect of a small change of current will now depend upon whether the current be increased or diminished. If the current be changed in the same way as it was altered last, a point A on the curve of magnetization will be reached; while if it be changed in the opposite direction, a point B will be reached. The slope of the line PA will be quite different from that of PB, and as these slopes represent the values of the coefficient of self-induction at the point P, it follows that this coefficient has always two distinct values whatever the state of magnetization and however that state has been attained. The curves obtained by Professor Ewing imply that the coefficient is greater for a change of current in the same direction as the last than for one in the opposite direction; for if P be any point on the ascending curve of magnetization, and P′ any point on the descending curve, the slopes PA and P′A′ correspond with changes of current in the same direction as those immediately preceding, and these slopes are steeper than those of PB and P′B′, which correspond with reverse changes of the magnetizing force.

If the current be changed between the two values corresponding with the points P and A, and the swings of the ballistic galvanometer observed for each alternation, the first fling will correspond with the slope PA, if the magnetization has been increased up to the point P. All the succeeding swings will, however, correspond with more gradual slopes, AC, because the magnetization owing to hysteresis will not return to the state P, but will proceed around a Ewing's cycle between the two points A and C, if the current be alternated between the two values considered. In the case of a transformer, two or three successively diminishing swings were often observed before the changes became cyclic. Whether on the ascending or descending portions of the magnetization-curve, the first fling should be greater than the succeeding

* "Experimental Researches in Magnetism," Phil. Trans. part ii. 1885.

ones if it corresponds with a change of current of the same kind as that immediately preceding.

The following experiments on the Ferranti field-magnets illustrate these remarks:—

Previous history.	Current Change.	Successive Swings.
C increased to +12 amperes, and diminished to 5·86.	5·86 to 3·56 3·56 to 5·86	−232 −205 +193 +192
C diminished to 0, and raised to 3·56.	3·56 to 5·86 5·86 to 3·56	+230 +197 +195 −209 −205
C increased to 4·86, and diminished to 3·13.	3·13 to 4·86 4·86 to 3·13	+149 +149 −151 −152
C increased to +13, and diminished to 4·86.	4·86 to 3·13 3·13 to 4·86	−180 −157 −157 +153 +152
C diminished to 0, and increased to 3·13.	3·13 to 4·86 4·86 to 3·13	+189 +153 +153 −162 −159

The swings succeeding the first are seen to be all very nearly equal. The negative swings are slightly greater than the positive ones, because the current flowing is smaller, the resistances in circuit larger, and the time-constant of discharge smaller. Although the amount discharged is the same for the two cases, the swing which measures it will be greater in that case for which the time-constant is the smaller.

13. Evidence of hysteresis was obtained in a very marked way from a 2 H.P. transformer lent to Professor Ayrton by Mr. Kapp. The coefficient of self-induction of the primary coil was measured when different currents were traversing the secondary. The magnetizing current (denoted by A) was obtained from accumulators, and varied up to 10 amperes. It was sent in both directions, and its value was read by an Ayrton and Perry ammeter. The coefficient was obtained either by comparison with a standard condenser by the swing-method already described or by the secohmmeter. The test-current used to obtain the coefficient was ·037 ampere for the swing method and ·01 ampere for the secohmmeter method. In the former case means were provided for sending the test-current round the primary in either direction at will, so that the swings obtained corresponded either with the slope PA (see fig. 1) or with the slope PB. If

the total current through the transformer-coils were alternated between two values corresponding with the points A and C, the slope AC was found not only to be less than the preceding slope PA but greater than the succeeding slopes CD, and in some cases several successively diminishing values were obtained. The coefficients corresponding with the slopes PA, AC, and CD will be respectively denoted by the letters L_p, L_r, and L_c. The values obtained could only be reproduced when the magnetic history of the iron was exactly repeated; and whatever the process of magnetization was, the numbers obtained at corresponding parts of successive cycles gradually diminished until the true cyclic values were obtained.

Thus the values obtained by the swing-method for the progressive coefficient L_p were ·087 secohm or ·192 secohm, according as the value of A had been diminished to zero from 1 ampere or from 6 amperes. When A was 1 ampere, values of L_p could be obtained varying between ·074 and ·130 secohm, according to the previous history, and simultaneously the values of the return coefficient L_r could be varied between ·032 and ·053 secohm. The values of the cyclic coefficient L_c (which were obtained by reversing the test-current several times before taking the swing) were, however, fairly constant, and were ·029 secohm when A was zero and ·016 secohm when A was 6 amperes.

In order to obtain a complete set of values for the coefficients L_p and L^c, it was necessary that the iron should go through the same magnetizing processes before any reading was taken. The magnetizing current was therefore diminished to -10 amperes before each observation, and then increased to the particular value of A at which the value of L was observed.

The annexed table indicates the results obtained by the swing method :—

A.	L_p.	L .
0	·243	·0290
2	·230	·0261
4	·083	·0193
5	......	·0177
6	·016	·0155
8	·013	·0120
10	·012	·0097

The values obtained with diminishing currents (*i. e.* for the curve P′A′) were slightly lower. They were arrived at by increasing the current to +10 amperes and then diminishing it to the value of A, at which L_p was required.

The values of L_c were much easier to obtain than those of L_p, for the slightest variation in the main magnetizing current A was sufficient to render it impossible to obtain L_p until the previous magnetizing processes were gone through again. Accumulators were used to provide a constant E.M.F.; but it was nevertheless found difficult to keep A sufficiently constant, so that the test-swings for L_p were taken as quickly as possible to prevent values intermediate between L_p and L_c being obtained. These considerations partially account for the unsatisfactory nature of the results obtained with the Ferranti field-magnets; for, at the time those experiments were made, the necessity of keeping the current quite steady until the swing measuring L was observed was not realized. The numbers given do not show the slight initial increase in the value of L_p for small increasing currents, but more recent experiments have done so.

The values of L_c were also determined by means of the secohmmeter of Professors Ayrton and Perry. This instrument necessarily only measures coefficients for cycles of magnetism. The value of A was varied from zero to 10 amperes positive, thence to 10 amperes negative, and thence to zero again. The value of each coefficient was determined for three different speeds, and the mean taken. The value of the coefficient appeared to diminish slightly as the speed increased. (This effect has been observed in several cases when the magnetic circuit is wholly or partially composed of iron, and is probably due to magnetic lag.) The coefficients L_c obtained for a complete cycle of current values are given in the four columns of the annexed table. The first two columns give the values obtained respectively for increasing and decreasing positive currents; the last two columns indicate those obtained for increasing and decreasing negative currents.

A.	Values of L_c.			
0	·0231	·0231		·0231
2	·0221	·0193	......	......
4	·0167	·0134	·0194	·0147
6	·0150	......	·0141	·0119
8	......	·0105	......	......
10		·0090	·0099	

The values obtained are slightly lower than those obtained by the swing-method. This is accounted for by the fact that the amplitude of the cycle is less. With the secohmmeter the test-current was only ·01 ampere, while it was ·037 ampere with the former method. The value of L_c will be smaller, the smaller the amplitude of the cycle. The secohmmeter is so sensitive that it would have been easy by means of it to measure L_c for cycles of very small amplitude.

Shape of Current Waves.

14. It is a matter of both theoretical and practical interest to determine the way in which currents change when the impressed electromotive forces are given at every instant of time and when the coefficients of self-induction are given in terms of the currents which are flowing. In calculations concerned with alternating-current problems it is usual to assume that the impressed electromotive forces are pure sine functions of the time, and that the coefficients of induction are constant quantities. These assumptions are more convenient than true. The coefficient of self-induction can be at once deduced from the curve of magnetization, and therefore can no more be expressed as a mathematical function of the current than electromagnetism itself. It therefore appears as if, in the treatment of such problems, graphical methods are to be preferred to analytical ones. If, in a simple circuit, the curve connecting the impressed electromotive force with time be given, together with the curve connecting magnetization with current, it is perfectly easy by purely graphical processes to obtain the curve connecting current with time. For we have

$$RC + \frac{dN}{dt} = E,$$

where E is the impressed electromotive force, N is the number

of lines of force enclosed by the circuit, R is the resistance of the circuit, C the current flowing, and t the time at which the different quantities are evaluated.

Now, if we can neglect Foucault currents, or possible magnetic lag, or anything analogous which would make N directly dependent on time, we may put

$$\frac{dN}{dt}=\frac{dN}{dC}\frac{dC}{dt},$$

where $\frac{dN}{dC}$ is the coefficient of self-induction, L. If we put

$$C_0=\frac{E}{R}, \quad T=\frac{L}{R},$$

we obtain

$$\frac{dC}{dt}=\frac{C_0-C}{T}.$$

Now C_0 is the value of the current which would be flowing if there were no self-induction ; and since E and R are given, it is possible to plot a curve having C_0 for ordinates and time for abscissæ. Moreover, since the curve connecting N with C is given, it is possible by graphical processes to find another connecting $\frac{1}{R}\frac{dN}{dC}$ or T with C. This curve (see Plate IX. fig. 3) should be plotted, with the values of C for ordinates and the values of T as abscissæ. It will be found convenient to plot T in the negative direction. The time-ratio T will be in seconds if L is in secohms and R in ohms. The two curves should of course be plotted to the same scale for current and time.

The construction follows immediately from the equation

$$\frac{dC}{dt}=\frac{C_0-C}{T},$$

and is as follows:—

Suppose P_1 (see Pl. IX. fig. 3) be the given initial point on the curve connecting C with time. Project it parallel to the axes to T_1 on the curve T and to Q_1 on the curve C_0. Project Q_1 parallel to the axis of abscissæ to R_1 on the current axis. Draw from P_1 a line parallel to R_1T_1, and choose a point P_2 on it not far from P_1. P_2 may be regarded as the next point on the current curve, and the process (which is indicated in Plate IX.) may be repeated to obtain a third point P_3, and so

on, until the whole curve connecting current with time is obtained.

15. The curves are very readily drawn on squared paper, and yield some remarkable results. If the impressed electromotive force E and the resistance R are constant quantities, the curve C_0 will be a straight line parallel to the axis of time cutting the current axis in a point R. The inclination of the line joining R to any point T on the curve T will represent the rate at which the current is increasing when it has the value corresponding with the point T. Now, if the self-induction is constant or nearly constant, this rate of increase will be great at first but will continually diminish as time goes on. If, however, the self-induction be very variable, the result may be quite different and dependent to a great extent on the value of C_0. Suppose the self-induction to increase at first, and then to diminish. Unless C_0 be very small, there will now be points of inflexion on the current curve. The current will increase very rapidly at first, slower afterwards, then more rapidly, and will finally attain its maximum slowly. If C_0 be such as to magnetize the iron far beyond saturation, this effect may be very marked, and the time taken for the current to rise to a small fraction of its final amount may exceed that taken to rise through the remainder. The writer is indebted to Prof. Silvanus Thompson for the information that this fact has actually been observed when accumulators have been connected in series with the field-magnets of a dynamo and with an Ayrton and Perry dead-beat ammeter. The needle has been noticed to move very slowly at first, and then with great rapidity through the larger portion of the ultimate deflection. The greater the value of C_0, the quicker will the current attain a given fraction of its final value. This may be roughly accounted for by the fact that the mean value of the coefficient is less the greater the value of the final current.

16. When the electromotive force E is alternating, the coefficient L will not only be variable but two-valued. It will depend not only on the value of the current, but also on whether the current is increasing or diminishing. The curve T will therefore consist of two parts—one for increasing current, and the other for decreasing current. Although E and C_0 may be pure sine functions, C will not be a simple sine

function if the coefficient L varies. Suppose a current curve C′, drawn on the assumption that there is a constant coefficient of self-induction equal to the mean of the different values. The true curve C will, roughly speaking, differ from the curve C′ in rising more rapidly when L is less than the average, and in rising less rapidly when L is more than the average. The general effect of the variation of the coefficient will therefore be to introduce ripples into the current curve. It is noteworthy that these ripples are dependent not so much on the speed as on the value of the current.

17. Plate IX. fig. 4 shows an example worked out on the basis of experiments actually made on the Kapp and Snell transformer already referred to. The resistance of the circuit is for simplicity assumed to be 1 ohm, so that the number which represents the coefficient of self-induction L_p in secohms also represents the time-ratio in seconds. The values of L_p given above were obtained for the primary coil when different currents A were traversing the secondary, and should be plotted, not with the values of A, but with the values of n_2A/n_1, where n_1/n_2 is the ratio of the turns on the primary to those on the secondary. This has not been done, as it merely alters the scale of current. The values of L_p differ slightly for increasing and decreasing currents; this has been disregarded, as in the particular cases taken the result would have been but very little altered. The electromotive force acting in the primary circuit (the secondary being open) is supposed to be a simple sine wave having a period of 0·16 second, and is represented by the curve C_0. The current curve obtained will depend for the first few alternations on the initial circumstances. The curves C_1, C_2, C_3, and C_4 represent the first half-wave for different initial values of the current. Of these, C_3 may be taken as the curve which periodically repeats itself and to which all the others will eventually come. It is not in appearance so markedly different from a sine curve as C_1 and C_2. This is because the impressed electromotive force is not sufficient to produce a current capable of magnetizing the iron beyond the saturation-point, at which the value of the coefficient of self-induction begins to diminish. Otherwise the current curve would have sharp peaks in it, as indicated in the curve C_1. This may have

something to do with the fact that it has been found best not to allow the iron of transformers to be magnetized beyond the saturation-point*.

18. It is usual to assume that the electromotive force given by alternating-current dynamos can be represented by a sine function of the time with sufficient accuracy for practical purposes. In the case of a Ferranti dynamo at the Central Institution, however, the approximation to a sine wave is not very good.

The field-magnets were excited with a small current by means of accumulators. A resistance of 100 ohms was placed in the circuit to reduce the time-constant when the current was made. The armature was used as a test-coil, and was coupled in series with a D'Arsonval galvanometer and suitable resistances. A pointer was attached to the spindle, and a scale to the framework of the machine, so that the phase of the armature could be accurately read. The test swings were taken both on making and on breaking the exciting current, and were found practically identical. The swings were taken for many different phases of the armature for one complete alternation. The results showed that the ordinates of the curve of electromotive force gradually differed from those of the most favourably drawn sine curve to the extent of ± 5 per cent.

The values of the maximum ordinates of the E.M.F. curve were also compared for one complete revolution of the armature. Five swings were taken in the neighbourhood of each maximum in order to determine its amount. The curious result came out that these maxima varied alternately between values denoted by the numbers 302 and 333. The eight positive maxima did not vary more than $\frac{1}{3}$ per cent. from the value 333, nor did the eight negative maxima vary more than this from the value 302. Since the area of the negative part of the curve must apparently be equal to that of the positive part, it follows that the curve of E.M.F. must be distinctly different from a sine curve. The result found in this case was not at all that which was looked for. It seemed probable that the values of the different maxima would vary

* See Mr. Gisbert Kapp's paper "On Alternate Current-Transformers," Proc. Soc. Tel. Engineers, Feb. 1888.

gradually, and only repeat themselves after a complete revolution of the machine. It was expected that the curve of E.M.F. would be of the nature of a compound sine curve, having as one term a sine of large amplitude and a period corresponding with the current alternations, and having as a second term a sine of small amplitude but long period corresponding with the revolutions of the machine. Professor Perry has pointed out that, if the wave of E.M.F. is a sine curve marked with minor ripples caused by the presence of subsidiary sine waves of smaller period, the effect of induction in the circuit is to flatten out the minor ripples so that the current produced is more nearly a true sine wave than the electromotive-force wave which is producing it. This is, however, only true on the assumption that the minor ripples have the shorter period. It is very possible that in some alternating-current dynamos there is, due to some want of symmetry, a ripple of small amplitude but having a long period corresponding with the revolutions of the dynamo. Whenever this occurs, the action of induction is to magnify the minor ripples in comparison with the main alternating wave, and the consequent current curve will, especially at high speeds, differ in a very marked degree from the E.M.F. curve of the machine. If the self-induction is a variable quantity, the current curves obtained will be still more rippled, and these ripples will be present at whatever speed the dynamo is revolved.

The well-known pulsations to which electric lights fed with alternating currents are subject seem to suggest that the currents obtained from the dynamos used are much less like sine waves than is generally assumed to be the case. These pulsations are so marked in the case of one important London installation that any quickly moving object, such as a walking-stick swayed rapidly to and fro presents, not a blurred image, as it would do under a continuous light, but several distinctly separate images, as if the light were at times very dim. The pulsations are too slow to be due to the ordinary alternations of the current, but might possibly be in time with the revolutions of the dynamo.

Condenser Discharges.

19. It does not appear to have been noticed that self-induction, although always delaying the rise or fall of currents, may sometimes hasten the discharge of a condenser. The current which discharges a condenser has both to rise and fall. Self-induction in the discharge-circuit delays the rise of current less than the fall, because the potential-difference of the condenser is high when the current increases and low when it diminishes; the rate of change of current depends, not only on the coefficient of self-induction, but also on the E.M.F. tending to cause the change. The well-known oscillations produced when the self-induction is large are simply due to the fact that the current is kept flowing too long. The general result is that the time of discharge is lessened if the self-induction is not too great.

If a condenser of capacity K, charged to a potential V_0, be discharged by a wire having a resistance R and coefficient of self-induction L, the potential of the condenser at any time t is given by one of the equations

$$V=\frac{V_0}{T_1-T_2}\left\{T_1 e^{-\frac{t}{T_1}}-T_2 e^{-\frac{t}{T_2}}\right\},$$

$$V=V_0 e^{-\frac{Rt}{2L}}\frac{\sin(\beta t+\gamma)}{\sin\gamma},$$

according respectively as L is less or greater than $\frac{KR^2}{4}$.

In these equations T_1 and T_2 are the two quantities whose sum is KR and whose product is KL; T_1 is the larger of the two, but is less than KR.

The values of β and γ are given by the equations

$$\beta^2=\frac{1}{KL}-\frac{R^2}{4L^2}, \quad \tan\gamma=\frac{2L\beta}{R}.$$

The time taken to discharge is proportional to T_1 or to $\frac{2L}{R}$, according as L is less or greater than $\frac{1}{4}KR^2$. Both these values are less than KR, provided $L<\frac{1}{2}KR^2$; so that the insertion of self-induction up to this value only hastens the discharge. The time will be a minimum when $L=\frac{1}{4}KR^2$;

and in this case $T_1 = \frac{1}{2}KR$. It is therefore possible to halve the time of discharge of a condenser by using a suitable amount of self-induction in the discharge-circuit.

A lightning discharge is essentially the same as that of a condenser, and it therefore seems likely that a small amount of self-induction in a lightning-conductor will only improve it. The value of K may be small, but that of R is large, since it quite possibly includes the resistance of the air-gap between the lightning-conductor and the charged cloud ; so that the product KR^2, which determines the allowable value of L, may be quite comparable with the self-induction of iron lightning-rods, and may indeed far exceed it.

Quite recently Dr. Oliver Lodge has shown experimentally that an iron conductor is better than a copper one for high-tension electric discharges, and has suggested that, owing to magnetic lag, iron may have less self-induction for such discharges, than in cases where time is an unimportant factor. The foregoing considerations offer quite a different explanation. Further experiments will no doubt be necessary to settle the question ; but in the meanwhile there seems to be no real basis for the idea that self-induction in a lightning-conductor is necessarily a disadvantage.

The writer, in conclusion, desires to thank Professor Ayrton for many valuable suggestions received while the experiments were being carried out.

XXXII. *On a Lecture Experiment for determining the Velocity of Sound.* *By* Prof. A. W. Rücker, *F.R.S.** (Abstract.)

The principle of the arrangement is that used by Fizeau in determining the velocity of light. A vibrating reed is used as the source of sound, and a sensitive flame as receiver.

A long U-shaped tube has its two ends placed near and parallel to the plane of a perforated disk, which is capable of rotating about an axis perpendicular to its own plane. The reed and sensitive flame occupy similar positions on the opposite side of the disk.

On rotating the disk, the sensitive flame flares or is quiescent

* Read November 12, 1887.

according as the time taken to travel the length of the tube is an even or an odd multiple of $\frac{I}{2n}$, where I is the time of one revolution, and n the number of holes in the disk.

XXXIII. *On a Form of Polariscope for Researches on the Polarization of the Sky.* *By* R. H. M. BOSANQUET, *F.R.A.S.** (Abstract.)

ITS chief feature is a compound prism of right- and left-handed quartz, which shows coloured bands with polarized light, whatever be the direction of the plane of polarization. It also forms a very sensitive object for polarimeters.

XXXIV. *On the Analogies of Influence-Machines and Dynamos.* *By* Prof. S. P. THOMPSON, *D.Sc.*† (Abstract.)

THE author pointed out that in nearly all influence-machines there are two stationary parts ("inductors") electrified oppositely, which are analogous to the field-magnets of dynamos, and a revolving part carrying "*sectors*," which correspond to the "*sections*" of an armature. To prevent ambiguity, Prof. Thompson proposes to call the inductors "*field-plates*," and the revolving parts as a whole an "*armature*."

In the Wimshurst machine both field-plates and armature rotate, and each act as field-plates and armature alternately.

In the two field-plate influence-machines there are four and sometimes six brushes. Two of these act as potential equalizers, two as field-plate exciters, and the remaining two (if any) are generally placed in the "discharge" or external circuit.

The Holtz machine having only four brushes, two serve the double purpose of potential equalizers and discharge-circuit; and this machine excites itself best when the discharging rods are in contact. In this respect it resembles a series dynamo, which only excites itself when the external circuit is closed, but on opening the circuit (say by inserting an arc-lamp) it

* Read November 12, 1887. † Read November 26, 1887.

produces remarkable effects. So in the Holtz machine, on separating the discharging knobs a shower of sparks results.

The Toepler machine (made by Voss), having six brushes, resembles a shunt dynamo, and excites itself best on open external circuit. Analogies were traced between Thomson's replenisher and the Griscom motor.

Armatures of influence-machines, as in dynamos, can be divided into Ring, Drum, Disk, and Pole armatures, and examples of each kind were mentioned. The "Clark Gas Lighter" is a good example of a Drum armature, and a diagram showing the internal arrangements was exhibited.

An example of an analogue to the Compound Dynamo was mentioned as existing at Cambridge, in the form of a Holtz machine, believed to have been modified by Clerk Maxwell.

Another analogue with dynamos is found in the displacement of the electric field when the armature is rotated, just as the magnetic field of a dynamo is shifted round in the direction of rotation.

Further analogies were traced between "critical velocity" of dynamos (which depends on the resistances in the circuit), below which they do not excite themselves, and a similar critical velocity of influence-machines ; *e. g.* in a Wimshurst or Voss machine, the potential equalizing circuit should have a low resistance if they are to excite themselves readily.

Self-exciting dynamos excite better when the iron is bad and retains the magnetism ; and influence-machines excite better when the field-plates are made of paper, or such substance as can well retain a residual charge.

Finally, an apparatus analogous to Thomson's "water-dropping accumulator" was exhibited, in which an electric current was generated by mercury falling down a tube between the poles of a magnet.

Prof. Ayrton pointed out an historical analogy between the invention of the influence-machine and the dynamo ; for, as Varley anticipated Thomson in the invention of the self-exciting influence-machine, so Hjorth anticipated Varley, Wheatstone, and Siemens in the invention of the self-exciting dynamo.

Dr. Fleming considered that, just as all influence-machines

may be looked upon as descendants of Volta's "Electrophorus," so dynamos are those of Faraday's "Coil and Magnet."

XXXV. *On the Rotation of a Copper Sphere and of Copper Wire Helices when freely suspended in a Magnetic Field. By* Dr. R. C. SHETTLE*. (Abstract.)

THE author exhibited the apparatus with which his experiments, "On the Supposed New Force," were made, the results of which were published in the 'Electrician,' vol. xix. Dr. Hofford has recently made similar experiments, using brass disks, and his results seem to point to "diamagnetic non-uniformity" of the disks as the cause of the phenomena he observed.

XXXVI. *On the Optical Properties of Phenyl-thio-carbimide. By* H. G. MADAN†. (Abstract.)

THIS body, derived from aniline, is a colourless liquid, density 1·35, and of high boiling-point, 222° C. The refractive indices for the A and G lines are 1·639 and 1·707 respectively. It is thus seen to be a highly refractive liquid, and to have about the same dispersive power as carbon bisulphide, whilst its use in prisms is unattended by many of the risks and inconveniences experienced with carbon bisulphide. The dispersion at the blue end of the spectrum is very marked. Being less mobile than carbon bisulphide it is less affected by convection-currents. The "refractive equivalent," calculated from its chemical constitution, differs considerably from the observed value, and this difference the author believes due to the presence of the phenyl radical and sulphur atom. A polarizing prism made on Jamin's plan, but using phenyl-thio-carbimide as the liquid, gives a fairly wide angular field (about 25°).

Mr. Hilger stated that there was no great need of liquid prisms now, for very dense flint-glass could be obtained with mean index of about 1·8. Dr. Perkin has recently supplied him with Canada balsam perfectly colourless, and which does

* Read December 10, 1887. † Read December 10, 1887.

not tarnish the polished faces of spar; hence one of the greatest objections to the use of Canada balsam in spar-polarizing prisms has been removed.

Dr. Gladstone pointed out that the constants for the phenyl radical and for sulphur atoms had been determined, and thought the calculated "refractive equivalent" obtained by including these would be much nearer the observed value than the one given by Mr. Madan.

XXXVII. *On the Optical Demonstration of Electrical Stress. By* Prof. A. W. RÜCKER, *F.R.S., and* Mr. C. V. BOYS*.

A NUMBER of Lecture-experiments were shown illustrating that electrical stress exists in the dielectric separating two charged bodies. The bodies were placed in carbon bisulphide between two crossed nicols, and on electrifying them by means of a Holtz machine, light passed through the analyzer. Two concentric cylinders gave a black cross on the screen similar to those seen on interposing a plate of some uniaxal crystal; and a model illustrating a charged Leyden jar was shown.

XXXVIII. *On a Compact Form of Reflecting-Galvanometer, Lamp, and Scale. By* G. L. ADDENBROOKE†. (Abstract.)

THE author exhibited this apparatus, designed as a portable commercial instrument, and also a modified Post-office Wheatstone-bridge. In the galvanometer the coils are easily removed or replaced by others, and loose wires for connecting the coils together are avoided; the required connexions being made through the screws, which secure the coils in position. The suspension is arranged so that the needle can be quickly taken out without breaking the fibre, and replaced by another for ballistic purposes if necessary. A simple liquid damping arrangement is provided. The lamp (a paraffin one) is enclosed in a copper cylinder, which also supports the scale. The height of the lamp and scale can be varied, and the beam of light can be directed at different altitudes by having the lens-tube mounted on pivots, the line joining them passing through the centre of the flame. The whole is very compact and portable.

* Read January 28, 1888. † Read March 10, 1888.